Program of the preliminary exam Wiskunde A
Valid from December 2018

The preliminary exam Wiskunde A is taken as a written examination with open questions. The exam time is 3 hours. Information about the exam dates and registration for these exams can be found on www.ccvx.nl.

The program of the preliminary exam Wiskunde A of the CCVW is based on the program wiskunde A of the Dutch VWO for 2019 as published on www.examenblad.nl. Compared to the Central Exam of VWO there are two important differences:

1. **At the preliminary exam Wiskunde A, graphing calculators** or any other ICT devices **may not be used**.
2. **The domain Statistics and Probability Theory is also tested at the preliminary exam Wiskunde A**. In the Dutch VWO exam, this domain is only tested at the School Exam.

The further determination of the VWO examination program on www.examenblad.nl is therefore not applicable.

In this document you will find
- The exam program
- Exam supplies
- The formula list that is printed on the exam
- Elaboration of the exam program in a list of concepts, characteristics and skills
- Overview of algebraic skills
- Recommended learning materials

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In preliminary exam all calculations must be performed algebraically, the use of a **graphing calculator** or a calculator with the possibility to calculate integrals is therefore **not allowed**.

The use of a standard calculator with exponential, logarithmic and trigonometric functions of a type similar to the Casio fx 82 series and the TI 30 series is permitted.
Exam program

This is a translation of the official program in Dutch. If there are any discrepancies between the text below and the Dutch version of this text, the Dutch version will prevail.

1. The candidate can analyse suitable problem situations in mathematical terms, solve them and translate the result back to the relevant context.

2. The candidate masters the mathematical skills appropriate to the examination program, including modelling and algebraizing, organizing and structuring, analytical thinking and problem solving, manipulating formulas, abstracting, and logical reasoning.

3. The candidate can perform calculations with numbers and variables, using arithmetic and algebraic basic operations and working with brackets.

4. The candidate masters the characteristic in graph, table and formula of first-degree functions, second-degree functions, power functions, exponential functions, logarithmic functions and sine functions.

5. The candidate can draw up and edit formulas and functional rules of combinations and assemblies of the types of functions mentioned in 4, draw the corresponding graphs, solve equations and inequalities with algebraic methods, thus without the use of a graphing calculator, and interpret the result in terms of a context.

6. The candidate can determine the derivative of first-degree functions, second-degree functions, power functions, exponential functions and logarithmic functions, use the rules for differentiation and describe the change behaviour of a function on the basis of the derivative. The use of the chain rule is limited to functions of the form \( k(x) = f(g(x)) \) in which \( f \) and \( g \) are single functions of the types mentioned in the previous sentence.

7. By means of the derived function, the candidate can calculate the position of the minima and the maxima of a graph by solving the associated equations algebraically and can determine the nature of these extremes by means of a sketch of the graph.

8. The candidate can recognize and describe the behaviour of a sequence and perform calculations on a sequence, especially for arithmetic and geometric sequences.

9. The candidate can structure counting problems and schematisation and use the result of this in calculations and reasoning.

10. The candidate can determine the mode and the median of a series of discrete numerical data and can use a simple calculator to calculate the mean and standard deviation of a population of limited size.

11. The candidate can use the concept of chance to determine the probability of a certain outcome or event in a random chance process by means of a probability (tree) diagram, of combinatorics and of laws of probabilities.

12. The candidate can set the probability distribution for random variables with a limited number of outcomes and calculate the expected value and the standard deviation.

13. The candidate can determine whether a binomial probability distribution is applicable in a given situation, indicate the parameters of this probability distribution and use the corresponding formulas to calculate probabilities, expected value and standard deviation.

14. The candidate can use the parameters of the normal distribution, i.e. the mean (or the expected value) and the standard deviation, and can use the rules of thumb for the location of 68% and 95% of the observations.
The candidate can determine the expected value and the standard deviation of the sum of two independent random variables and can determine the expected value and the standard deviation of the sum and the average of the results of n independent repetitions of the same random experiment.

The candidate can set up a statistical testing procedure from a suitable problem situation and thereby
- indicate whether it concerns a test of the mean or a proportion test
- formulate the null hypothesis
- indicate whether the procedure must be carried out left-sided, right sided or two-sided
- indicate how a given sample result must be converted to a p-value
- interpret the given value of a p-value
  calculate the boundaries of the rejection region (the critical region) with the table in the formula list and interpret the given sample result using these boundaries.

The translation of the next section is still in progress.

Tentamenbenodigdheden
Naar het tentamen moet u meenemen:

- Identiteitsbewijs
  Paspoort, rijbewijs, Europese ID kaart, verblijfsdocument
- Schrijfgerei: pen
  *Een potlood mag alleen gebruikt worden voor het tekenen van grafieken.*
- Liniaal of geodriehoek
- Rekenmachine met exponentiële, logaritmische en goniometrische functies
  *Grafische rekenmachines en rekenmachines met de mogelijkheid om integralen te berekenen zijn niet toegestaan."
- Een horloge (geen smartwatch) of een klok (niet de klok van uw telefoon)
- Eten en drinken

Zorg ervoor dat u de juiste rekenmachine meeneemt. Als u alleen een grafische rekenmachine bij u heeft, dan zult u het tentamen zonder rekenmachine moeten maken.
# Formula list
This list will be printed on the last pages of the preliminary exam Wiskunde A

## Quadratic equations
The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

## Differentiation

<table>
<thead>
<tr>
<th>Rule</th>
<th>function</th>
<th>derivative function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum rule</td>
<td>$s(x) = f(x) + g(x)$</td>
<td>$s'(x) = f'(x) + g'(x)$</td>
</tr>
<tr>
<td>Product rule</td>
<td>$p(x) = f(x) \cdot g(x)$</td>
<td>$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$</td>
</tr>
<tr>
<td>Quotient rule</td>
<td>$q(x) = \frac{f(x)}{g(x)}$</td>
<td>$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$</td>
</tr>
<tr>
<td>Chain rule</td>
<td>$k(x) = f(g(x))$</td>
<td>$k'(x) = f'(g(x)) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$</td>
</tr>
</tbody>
</table>

## Logarithms

<table>
<thead>
<tr>
<th>Rule</th>
<th>conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{log}_a a + \text{log}_a b = \text{log}_a ab$</td>
<td>$g &gt; 0, \ g \neq 1, \ a &gt; 0, \ b &gt; 0$</td>
</tr>
<tr>
<td>$\text{log}_a a - \text{log}_a b = \text{log}_a \frac{a}{b}$</td>
<td>$g &gt; 0, \ g \neq 1, \ a &gt; 0, \ b &gt; 0$</td>
</tr>
<tr>
<td>$\text{log}_a a^p = p \cdot \text{log}_a a$</td>
<td>$g &gt; 0, \ g \neq 1, \ a &gt; 0$</td>
</tr>
<tr>
<td>$\text{log}_a a = \frac{\text{log}_a a}{\text{log}_a g}$</td>
<td>$g &gt; 0, \ g \neq 1, \ a &gt; 0, \ p &gt; 0, \ p \neq 1$</td>
</tr>
</tbody>
</table>

## Arithmetic and geometric sequences

<table>
<thead>
<tr>
<th>Arithmetic sequence:</th>
<th>$\text{Sum} = \frac{1}{2} \cdot \text{number of terms} \cdot (u_e + u_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric sequence:</td>
<td>$\text{Sum} = \frac{u_{i+1} - u_e}{r - 1}$ (for $r \neq 1$)</td>
</tr>
</tbody>
</table>

*In both formulas:* $e =$ number first term of the sum; $l =$ number last term of the sum
Probability

If $X$ and $Y$ are any random variables, then: $E(X + Y) = E(X) + E(Y)$
If furthermore $X$ and $Y$ are independent, then: $\sigma(X + Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:
For $n$ independent repetitions of the same experiment where the result of each experiment is a random variable $X$, the sum of the results is a random variable $S$ and the mean of the results is a random variable $\bar{X}$.

- $E(S) = n \cdot E(X)$
- $\sigma(S) = \sqrt{n} \cdot \sigma(X)$
- $E(\bar{X}) = E(X)$
- $\sigma(\bar{X}) = \frac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If $X$ has a binomial distribution with parameters $n$ (number of experiments) and $p$ (probability of success at each experiment), then

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k} \quad \text{with} \quad k = 0, 1, 2, ..., n$$

- Expected value: $E(X) = np$
- Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1 - p)}$

$n$ and $p$ are the parameters of the binomial distribution.

Normal Distribution

If $X$ is a normally distributed random variable with mean $\mu$ and standard deviation $\sigma$, then

$$Z = \frac{X - \mu}{\sigma} \quad \text{has a standard normal distribution and} \quad P(X < g) = P \left(Z < \frac{g - \mu}{\sigma}\right)$$

$\mu$ and $\sigma$ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic $T$ is normally distributed with mean $\mu_T$ standard deviation $\sigma_T$, the boundaries of the rejection region (the critical region) are:

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>left sided</th>
<th>right sided</th>
<th>two sided</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>$g = \mu_T - 1.645\sigma_T$</td>
<td>$g = \mu_T + 1.645\sigma_T$</td>
<td>$g_1 = \mu_T - 1.96\sigma_T$</td>
</tr>
<tr>
<td>0.01</td>
<td>$g = \mu_T - 2.33\sigma_T$</td>
<td>$g = \mu_T + 2.33\sigma_T$</td>
<td>$g_1 = \mu_T - 2.58\sigma_T$</td>
</tr>
</tbody>
</table>
Elaboration of the exam program

Below, the exam program is further elaborated in a list of concepts, properties and skills. This list is intended as a support in preparing for the preliminary exam, but not as a replacement for the exam program. Although this list has been compiled with the greatest possible care, it may therefore occur that an exam question is not dealt with in this list.

<table>
<thead>
<tr>
<th>Concept / property / skill</th>
<th>Remark / explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Various calculations</strong></td>
<td></td>
</tr>
<tr>
<td>Computations with percentages; percentual change</td>
<td>$\frac{\text{New} - \text{Old}}{\text{Old}} \times 100%$</td>
</tr>
<tr>
<td>Scientific notation</td>
<td>$0.0000123 = 1.23 \times 10^{-5}$</td>
</tr>
<tr>
<td>Computations with fractions</td>
<td></td>
</tr>
<tr>
<td>Ratios</td>
<td></td>
</tr>
<tr>
<td>Computations with variables:</td>
<td>In a table of ratios the cross products are equal</td>
</tr>
<tr>
<td>Eliminating parentheses</td>
<td></td>
</tr>
<tr>
<td>Solve for a variable</td>
<td>$y = \frac{1}{x + 1} \Leftrightarrow x = \frac{1}{y} - 1$</td>
</tr>
<tr>
<td>Computation with units</td>
<td>The train runs 12 km in 5 minutes. That is 2.4 km/minute Or 40 m/s Or 144 km/h</td>
</tr>
</tbody>
</table>
## Powers, roots and radicals

- \( a^n \) is a power with base number \( a \) and exponent \( n \)
- \( \sqrt[n]{a} \) is the number of which the \( n \)-th power equals \( a \)

\( n \) is a rational number (a fraction)

\( n = 2, 3, 4, 5, ... \)

If \( n \) is even, then \( a \geq 0 \) and \( \sqrt[n]{a} \geq 0 \)

### Special exponents

- \( a^2 = a \cdot a \)
- \( a^3 = a \cdot a \cdot a \) etc.

\( a^1 = a \)

\( a^0 = 1 \)

\( a^{-n} = \frac{1}{a^n} = \left( \frac{1}{a} \right)^n \)

\( a^{\frac{1}{n}} = \sqrt[n]{a} \)

\( a \neq 0 \)

\( a \neq 0 \)

\( a^{-1} = \frac{1}{a^1} = \frac{1}{a} \)

\( n = 2, 3, 4, 5, ... \)

If \( n \) is even, then \( a \geq 0 \) and \( a^{\frac{1}{n}} \geq 0 \)

\( a^{\frac{1}{2}} = \sqrt[2]{a} = \sqrt{a} \)

### Calculation rules for powers

\( a^m \cdot a^n = a^{m+n} \)

\( \frac{a^m}{a^n} = a^{m-n} \)

\( (a^m)^n = a^{m \cdot n} \)

\( a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left( \sqrt[n]{a} \right)^m \)

\( a^{\frac{1}{n}} = \sqrt[n]{a} = \sqrt[2]{a} = \sqrt{a} \)

\( (a \cdot b)^n = a^n \cdot b^n \)

\( (a + b)^2 \neq a^2 + b^2 \)

\( \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \)
<table>
<thead>
<tr>
<th>Straight lines and linear relations</th>
<th>Power relations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General equation:</strong> ( mx + ny = c )</td>
<td>The line with equation ( 3x + 4y = 12 ) passes through (4,0) and (0,3)</td>
</tr>
<tr>
<td>( n = 0 ) yields a vertical line</td>
<td>( 3x + 0 \cdot y = 12 ) yields ( x = 4 )</td>
</tr>
<tr>
<td>( m = 0 ) yields a horizontal line</td>
<td>( 0 \cdot x + 4y = 12 ) yields ( y = 3 )</td>
</tr>
<tr>
<td>The equation of a non-vertical line can also be written in the form ( y = ax + b )</td>
<td>( 3x + 4y = 12 ) yields ( y = -\frac{3}{4}x + 3 )</td>
</tr>
</tbody>
</table>
| Slope of the line through the points \( A(x_A, y_A) \) and \( B(x_B, y_B) \): \[
\frac{\Delta y}{\Delta x} = \frac{y_B - y_A}{x_B - x_A}
\] | This is equal to \( a \) in the formula 
\( y = ax + b \) |
| Drawing up an equation with the slope | Substitute the slope and the coordinates of a point in the formula \( y = ax + b \) |
| Computing the intersection of two straight lines | Eliminate \( y \) from the system of two equations |
| Interpolation and extrapolation | |
| Direct proportionality and inverse proportionality | |

<table>
<thead>
<tr>
<th>Power relations</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = ax^n ) with ( n ) even</td>
<td>The graph has a vertex in (0,0)</td>
</tr>
<tr>
<td>( y = ax^n ) with ( n ) odd</td>
<td>( (0,0) ) is the punt of symmetry of the graph</td>
</tr>
<tr>
<td>Horizontal shift</td>
<td>( y = 3(x - 4)^2 + 5 ) is obtained from the graph of ( y = x^2 ) by tripling the ( y )-coordinates, followed by shifting the graph 4 units to the right and 5 units upwards. This results in an upwards opening parabola with vertex (4,5).</td>
</tr>
<tr>
<td>Vertical shift</td>
<td></td>
</tr>
<tr>
<td>Rescaling in vertical direction</td>
<td></td>
</tr>
</tbody>
</table>
### Growth formulas

**Exponential growth with initial value \( b \) and growth factor \( g \)**

The graph of \( N(t) = b \cdot g^t \) is increasing for \( g > 1 \) and decreasing for \( 0 < g < 1 \).

<table>
<thead>
<tr>
<th>Growth factor and growth percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Converting a growth percentage to a different unit of time</td>
</tr>
<tr>
<td>Doubling time for ( g &gt; 1 )</td>
</tr>
<tr>
<td>Half-life for ( 0 &lt; g &lt; 1 )</td>
</tr>
</tbody>
</table>

**Other growth formulas**

\[
N(t) = a(1 - g^t)
\]

\[
N(t) = \frac{a}{1 + c \cdot g^t}
\]

For \( 0 < g < 1 \), for both formulas the graphs are increasing to \( a \), the boundary (the saturation level).

### Logarithms

\( y = g^\log(x) \) means \( g^y = x \)

\[
g^\log(1) = 0; \quad g^\log(g) = 1; \quad g^\log(g^n) = n
\]

\[
\log(x) = 10^\log(x)
\]

The graph is increasing for \( g > 1 \) and decreasing for \( 0 < g < 1 \).

\[
g > 0, \ g \neq 1; \ x > 0
\]

\[
\log(1) = 0; \ \log(10) = 1; \ \log(100) = 2
\]

If \( x \) approaches 0, the graphs approach the \( y \)-axis.

### Calculation rules for logarithms

\[
g^\log(a) + g^\log(b) = g^\log(a \cdot b)
\]

\[
g^\log(a) - g^\log(b) = g^\log\left(\frac{a}{b}\right)
\]

\[
g^\log(a^n) = n \cdot g^\log(a)
\]

\[
g^\log(a) = \frac{\log(a)}{\log(g)}
\]

These follow from the calculation rules for powers

This holds for all numbers \( n \)

Often used with \( p = 10 \) to compute logarithms with the log key of a calculator.
Differentiation: the derivatives of standard functions

\[ f(x) = a \quad \text{yields} \quad f'(x) = 0 \]
\[ f(x) = ax \quad \text{yields} \quad f'(x) = a \]
\[ f(x) = ax^n \quad \text{yields} \quad f'(x) = n \cdot x^{n-1} \]
\[ f(x) = e^x \quad \text{yields} \quad f(x) = e^x \]
\[ f(x) = g^x \quad \text{yields} \quad f'(x) = g^x \cdot \ln (g) \]
\[ f(x) = \ln (x) \quad \text{yields} \quad f'(x) = \frac{1}{x} \]

The derivative is also noted as \( \frac{dy}{dx} \)

From the sum rule we get:
\[ f(x) = ax + b \quad \text{yields} \quad f'(x) = a \]

This also holds when \( n \) is a fraction or a negative number.
\[ e \approx 2,71828 \]
\[ \ln(g) = \log(g) \quad (g > 0) \]

From the constant factor rule we get:
\[ f(x) = g \log(x) \quad \text{yields} \quad f'(x) = \frac{1}{x \ln (g)} \]

Rules for differentiation of combinations of functions

Constant factor rule
Sum rule
Product rule
Quotient rule
Chain rule

\[ g(x) = c \cdot f(x) \quad \text{yields} \quad g'(x) = c \cdot f'(x) \]

The other rules are in the formula list

Applications of the derivative

\[ f'(a) \] is the slope of the (tangent to the) graph of \( f \) in point \((a, f(a))\)

The graph is increasing if \( f'(x) > 0 \) and decreasing if \( f'(x) < 0 \)

In a minimum or a maximum, \( f'(x) = 0 \)
Minima and maxima together are called extremes
The points where the graph has a minimum or a maximum are called the vertices of the graph.

Minimal and maximal rate of growth

The points where \( f \) could have a maximum or a minimum are determined by solving \( f'(x) = 0 \).
In principle you further have to check in which of these points the graph has a minimum and in which point the graph has a maximum. This often follows from the wording of the question.

The derivative (the rate of growth) can be differentiated as well
### Sequences

<table>
<thead>
<tr>
<th>Formula Type</th>
<th>Description</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct formula</td>
<td>Computes a term based on its number</td>
<td>Computes a term based on its number</td>
</tr>
</tbody>
</table>
| Recursive formula | Computes a term based on the previous term | Recursive formula: \( u_{n+1} = u_n + a \)  
Direct formula: \( u_n = u_0 + a \cdot n \)  
Or: \( u_n = u_1 + a \cdot (n - 1) \) |
| Arithmetic sequence |  | Recursive formula: \( u_{n+1} = r \cdot u_n \)  
Direct formula: \( u_n = r^n \cdot u_0 \)  
Or: \( u_n = r^{n-1} \cdot u_1 \) |
| Geometric sequence |  | See the formula list |
| Sum formula of an arithmetic sequence |  |  |
| Sum formula of a geometric sequence |  |  |
| Computing the number of a term |  | This is done by putting the direct formula equal to the value of the term. |

### Sinus functions

<table>
<thead>
<tr>
<th>Formula Type</th>
<th>Description</th>
</tr>
</thead>
</table>
| General formula: | \( f(x) = A + B \cdot \sin(C \cdot (x - D)) \)  
The graph is a sinusoid |
|  |  |
| Read the equilibrium, the amplitude, the period and a starting point from the graph |
| Determine the equilibrium, the amplitude, the period and a starting point from the formula |
| Use the period and the symmetry around the minimum and the maximum |
|  |  |
| \( A = \text{equilibrium} \)  
\( |B| = \text{amplitude} \)  
\( C = \frac{2\pi}{\text{period}} \)  
If \( B > 0 \) the graph passes the equilibrium increasing in point \((D, A)\) (a starting point) |
# Descriptive Statistics

*This subject is only dealt with superficially in the preliminary Wiskunde A*

<table>
<thead>
<tr>
<th>Mode, median, mean and standard deviation for a series of discrete numerical data</th>
<th>Mean and standard deviation may be computed with the statistic functions of an ordinary calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean and standard deviation for a frequency distribution</td>
<td></td>
</tr>
</tbody>
</table>

## Counting systematically

*A.k.a. combinatorics*

<table>
<thead>
<tr>
<th>Visualisation</th>
<th>E.g. tree diagram, contingency table, venn diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication rule</td>
<td>I AND II</td>
</tr>
<tr>
<td>Addition rule</td>
<td>I OR II (exclusive)</td>
</tr>
<tr>
<td>With or without repetition</td>
<td>The order of the objects matters</td>
</tr>
<tr>
<td>Permutations, factorials</td>
<td>The order of the objects does not matter</td>
</tr>
<tr>
<td>Combinations</td>
<td></td>
</tr>
</tbody>
</table>
| Pascal’s triangle | The number of codes with 5 A’s and 3 B’s is on the 8th row of Pascal’s triangle and is equal to  
\[
\binom{8}{3} = \binom{8}{5} = \frac{8!}{3! \cdot 5!} = 56
\] |
| Combination of techniques | The number of codes with 3 A’s, 4 B’s and 5 C’s is given by  
\[
\binom{12}{3} \cdot \binom{9}{4}
\]  
(and by  
\[
\binom{12}{5} \cdot \binom{7}{4}
\]) |
**Probability**

An event is a part of all possible outcomes of a random experiment. Is often indicated with a capital letter, such as G, A and B.

Laplace’s definition of probability for random experiments with equally likely outcomes:

\[ P(G) = \frac{\text{number of outcomes satisfying } G}{\text{total number of outcomes}} \]

Empiric probabilities are determined with relative frequencies. Theoretical probabilities are often determined by counting systematically.

Conditional probabilities are often determined with a contingency table.

Independent events

Multiplication rule for independent events

Addition law for mutually exclusive events

Complement rule

Application of these laws and rules for combined random experiments

**Random variables**

If the results of a random experiment are numbers, you can write down the corresponding probabilities using a random variable. If \( X \) is the number of eyes on a roll with a die, then we note the probability of 4 eyes with \( P(X = 4) \).

The probability distribution is a list of all possible outcomes of a random variable with the associated probabilities. The probability distribution is often noted in a table, for example:

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X = x) )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{6} )</td>
</tr>
</tbody>
</table>

Expected value (or expectation); Expected value of a sum = sum of the expected values.

\[ E(X) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \cdots + 6 \cdot \frac{1}{6} \]

\[ E(X + Y) = E(X) + E(Y) \]

Standard deviation

Standard deviation of the sum and the mean of independent random variables; \( \sqrt{n} \)-law

See the formula list.
<table>
<thead>
<tr>
<th><strong>The binomial distribution</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bernoulli trial</td>
<td>Outcome: “success” or “failure”</td>
</tr>
<tr>
<td>Binomial random experiment</td>
<td>Parameters:</td>
</tr>
<tr>
<td></td>
<td>( n = ) number of Bernoulli trials</td>
</tr>
<tr>
<td></td>
<td>( p = P(\text{success}) ) at each trial</td>
</tr>
<tr>
<td>Single probabilities (probability of ( k ) successes)</td>
<td>( P(X = k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k} )</td>
</tr>
<tr>
<td>Cumulative probabilities</td>
<td>( P(X \leq k); P(X \geq k); P(k \leq X \leq l) )</td>
</tr>
<tr>
<td></td>
<td>Only for a sum of at most 3 simple probabilities</td>
</tr>
<tr>
<td>Expected value</td>
<td>( E(X) = np )</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>( \sigma_X = \sqrt{np(1-p)} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>The normal distribution</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>The normal distribution and the Gaussian curve as an approximation of a frequency distribution</td>
<td>Parameters:</td>
</tr>
<tr>
<td></td>
<td>( \mu ) or ( \mu_X = ) mean (expected value)</td>
</tr>
<tr>
<td></td>
<td>( \sigma ) or ( \sigma_X = ) standard deviation</td>
</tr>
<tr>
<td>Rules of thumb</td>
<td>50% of the observations are below the mean</td>
</tr>
<tr>
<td></td>
<td>68% of the observations are between ( \mu - \sigma ) en ( \mu + \sigma );</td>
</tr>
<tr>
<td></td>
<td>95% of the observations are between ( \mu - 2\sigma ) en ( \mu + 2\sigma )</td>
</tr>
<tr>
<td>Computing ( \mu ) and/or ( \sigma ) using the rules of thumb</td>
<td>E.g. given that 84% of a population weighs more than 80 kg and 2.5 of this population weighs more than 110 kg.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Normal approximation</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A discretely distributed random variable can often be approached with a normal distribution</td>
<td>For a binomially distributed random variable, this is the case when ( np \geq 5 )</td>
</tr>
<tr>
<td>The mean and standard deviation of the normal approach are the expected value and the standard deviation of the discrete random variable.</td>
<td>Thus, for the normal approximation ( Y ) of a binomially distributed random variable with parameters ( n ) and ( p ), we get: ( \mu_Y = np ) and ( \sigma_Y = \sqrt{np(1-p)} )</td>
</tr>
<tr>
<td>Continuity correction</td>
<td>( P(X \leq k) = P(X &lt; k + \frac{1}{2}) )</td>
</tr>
<tr>
<td><strong>Hypothesis testing</strong></td>
<td><strong>Proportion test (binomial test) with null hypothesis ( H_0: p = p_0 )</strong> or mean test (normal test) with null hypotheses ( H_0: \mu = \mu_0 )</td>
</tr>
<tr>
<td>------------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Deriving the type of test procedure and the accompanying null hypothesis from a description</td>
<td>Left sided, right sided or two sided</td>
</tr>
<tr>
<td>Deriving the alternative hypothesis</td>
<td>The parameters for a binomial test are ( n ) and ( p_0 ), the parameters for a normal test are ( \mu_0 ) and ( \sigma ), use the ( \sqrt{n} )-law to compute ( \sigma )</td>
</tr>
<tr>
<td>Deriving the test statistic and its parameters</td>
<td>Usually ( \alpha = 0.05 ), sometimes ( \alpha = 0.01 )</td>
</tr>
<tr>
<td>Level of significance</td>
<td>Since the use of graphing calculators and tables is forbidden, the p-value usually will be given at the preliminary exam Wiskunde A</td>
</tr>
<tr>
<td>Decision based on a p-value</td>
<td>The boundaries for a normal test can be calculated with the formulas from the formula list. If ( np_0 \geq 5 ), you can also use these formulas with a binomial test. In this situation, you do not have to use the continuity correction</td>
</tr>
<tr>
<td>Decision by comparing the sample result with the boundary of the rejection region (the critical region)</td>
<td></td>
</tr>
</tbody>
</table>
**Algebraic skills**

Below an overview of algebraic skills that the candidates for the examination mathematics A of the CCVW must master. Though this list has been compiled with the utmost care too, it may occur that a skill that is included the exam program, is not included in this list.

<table>
<thead>
<tr>
<th>Skill</th>
<th>Remark / explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solving standard equations</strong></td>
<td></td>
</tr>
<tr>
<td>First degree</td>
<td></td>
</tr>
<tr>
<td>Quadratic</td>
<td>$ax + b = px + q$</td>
</tr>
<tr>
<td><em>With the quadratic formula</em></td>
<td>$ax^2 + bx + c = 0$</td>
</tr>
<tr>
<td><em>If possible, factorising can be used as well</em></td>
<td></td>
</tr>
<tr>
<td>Power equations with positive even exponents</td>
<td>$x^n = a \iff x = \pm \sqrt[n]{a} \quad (a \geq 0)$</td>
</tr>
<tr>
<td>Power equations with positive odd exponents</td>
<td>$x^n = a \iff x = \sqrt[n]{a}$</td>
</tr>
<tr>
<td>Other power equations</td>
<td>$x^n = a \iff x = a^{\frac{1}{n}} \quad (a \geq 0)$</td>
</tr>
<tr>
<td>Equations with roots</td>
<td>$\sqrt{x} = a \iff x = a^2 \quad (x \geq 0; \ a \geq 0)$</td>
</tr>
<tr>
<td>Solving suitable exponential and logarithmic equations exactly</td>
<td>$^a \log(g^n) = a \iff g^n = g^a \iff n = a$</td>
</tr>
<tr>
<td>Solving exponential equations using logarithms</td>
<td>$g^x = a \iff x = \frac{1}{a} \log(a)$</td>
</tr>
<tr>
<td><strong>Solving inequalities</strong></td>
<td><strong>When solving inequalities graphically, the intersections of the graphs have to be computed algebraically</strong></td>
</tr>
<tr>
<td><strong>Solving systems of equations</strong></td>
<td><strong>By elimination and/or substitution</strong></td>
</tr>
<tr>
<td><strong>Setting the equation of a straight line</strong></td>
<td><strong>Determine the slope and substitute this together with to coordinates of one point into one of the standard formula</strong></td>
</tr>
<tr>
<td><strong>Rewriting equations and formulas</strong></td>
<td><strong>E.g. to rewrite an equation into a standard equation</strong></td>
</tr>
<tr>
<td>------------------------------------</td>
<td>-----------------------------------------------</td>
</tr>
</tbody>
</table>
| Splitting an equation              | \( A \cdot B = 0 \Leftrightarrow A = 0 \lor B = 0 \)  
|                                    | \( A^2 = B^2 \Leftrightarrow A = \pm B \)  
|                                    | \( A \cdot B = A \cdot C \Leftrightarrow A = 0 \lor B = C \) |
| Factorising using a common factor   | \( A \cdot B + A \cdot C = A(B + C) \) |
| Expanding parentheses              | \((A + B)(C + D) = AC + AD + BC + BD\) |
| Operations with fractions          | Addition (also of fractions with unlike denominators) |
|                                    | Multiplication and division |
|                                    | Crosswise multiplication: |
|                                    | \( \frac{A}{B} = \frac{C}{D} \Leftrightarrow AD = BC \land BD \neq 0 \) |
| Operations with roots              | \( \sqrt{A} = B \Leftrightarrow A = B^2 \land A \geq 0 \) |
| Applying calculation rules         | Like those for powers and logarithms |

| **Combining formulas**              | **If e.g. the price per piece \( p \) and the cost \( c \) both depend on the quantity \( q \), then you can derive formulas for the revenue: \( r = pq \) and for the profit:  
|                                    | \( \pi = r - c = pq - c \) |
|                                    | Take care of the right use of parentheses when substituting formulas for \( p \) and \( c \). |

<table>
<thead>
<tr>
<th><strong>Rewriting formulas with logarithms</strong></th>
<th><strong>Use the fourth calculation rule</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Change of base</td>
<td><strong>Use ( Y = \log^g(X) \Leftrightarrow g^Y = X )</strong></td>
</tr>
<tr>
<td>Solving an exponential or logarithmic equation for a variable</td>
<td><strong>Use ( B = g^b; \ G = g^a )</strong></td>
</tr>
<tr>
<td>Converting ( g \log(N) = at + b ) into a formula of the form ( N = B \cdot G^t ) and vice versa</td>
<td><strong>( A = g^a )</strong></td>
</tr>
<tr>
<td>Converting ( g \log(N) = a + b \cdot g \log(t) ) into a formula of the form ( N = A \cdot t^b ) and vice versa</td>
<td><strong>( B = g^b; \ G = g^a )</strong></td>
</tr>
</tbody>
</table>
Recommended learning materials

Unfortunately, as far as we know there are no textbooks in English covering the full program of the Dutch VWO exam wiskunde A. However, there is a textbook that covers most of the calculus topics (functions and derivatives):


From this book you can skip the sections that cover antidifferentiation and integrals.

The topics Combinatorics, Statistics and Probability theory are covered in many textbooks on basic statistics. Be sure that the topics indicated in this document are covered in a book before you buy it!

There is also a wide range of materials available online, for instance the Kahn academy. You can use Google to find more information on a certain topic.

And last but not least, check the example exams on [www.ccvx.nl](http://www.ccvx.nl)