CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Example Exam Wiskunde A

Date:	Autumn 2018
Time:	3 Hours
Questions:	6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in reduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put in your bag.

Points that can be scored for each question:						
Question	1	2	3	4	5	6
a	4	2	4	3	5	3
b	4	3	5	4	5	6
c	4	6	2	3	3	
d	4		4		4	
e	4		4		4	
Total	20	11	19	10	21	9

$\text{Grade} = \frac{\text{total points scored}}{10} + 1$

You will pass the exam if your grade is at least 5.5 .

Question 1 – Algebraic computations

Source: CCVX entrance exam wiskunde A April 2018 (adjusted)

When you are asked to perform a computation ***algebraically****, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed as well in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like* $\sqrt{2}$ *and* $\log(3)$.

Solve the following equations algebraically. If there is a square root or a logarithm in your answer, give the answer rounded to three digits behind the decimal dot.

4pt a $3x^2 - 7x + 2 = 0$

4pt b $2x^4 - 4x^8 = 0$

4pt c $2 \cdot 4^x - 4 \cdot 8^x = 0$

4pt d $2 \cdot 5^x - 30 = 0$

The function f is given by $f(x) = \sqrt{4x - 7}$.
P and *Q* are the intersections of the graph of f and the line $y = \frac{1}{2}x + 1$.

4pt e Compute algebraically the coordinates of points *P* and *Q*.

Extra item

4pt f Compute algebraically the slope of $f(x) = \sqrt{4x - 7}$ in point $(2, 1)$.

Question 2 – The maximum return of an investment

Source: CCVX entrance exam wiskunde A April 2018 (adjusted)

Pastry chef Peter has developed a recipe for banana pie. Since he does not have money of his own, he is looking for an investor to finance the production of these banana pies. In order to convince this investor, he asks his friend accountant Anton to draw up a model that will predict the possible profit of the investor.

Anton presents the following model:

- The fixed cost for producing the banana pies is 250 euro.
- The variable cost per produced pie is 2 euro.
- If the selling price is set by the formula $p(q) = 14.00 - 0.05q$ (with $p(q)$ in euros and q the number of produced pies) all produced pies will be sold.

Peter indeed uses the formula $p(q) = 14.00 - 0.05q$ to determine the selling price and he wants to know how many pies he must produce to maximize the profit. Anton explains that the profit in euro's is given by the formula

$$Pr(q) = -0.05q^2 + 12q - 250$$

2pt **a** Use the relation $Profit = revenue - total\ cost$ to verify that the formula given above is correct.

3pt **b** Use the derivative function to determine the number of produced pies at which the profit is maximal.

To the investor, not only the profit, but also the return on his investment is of importance. This is given by the formula

$$Return\ on\ investment = \frac{profit}{investment} \times 100\%$$

The return (in %) for the investor in Peter's banana pies as a function of the number of produced pies q is thus given by

$$R(q) = \frac{-5q^2 + 1200q - 25\,000}{250 + 2q}$$

6pt **d** Compute algebraically the number of produced pies at which the return on investment is maximal for the investor.

Question 3 – Pear grower Paul

Source: CCVX entrance exam wiskunde A April 2018 (adjusted)

Pear grower Paul is very proud of his harvest this year. His Conference pears are tastier and heavier than ever. Because there cannot be a dispute about taste, in this question we focus on the weight of the pears. This is normally distributed with a mean of 190 grams and a standard deviation of 7 grams.

At an examination 10 pears are randomly chosen from the harvest of Paul.

4pt a Compute the probability that these 10 pears all weigh more than 190 grams.

The pears are packed in boxes of 36 pieces. The empty weight of these boxes is normally distributed with a mean of 500 grams and a standard deviation of 20 grams.

4pt b Compute the mean and the standard deviation of the total weight of a box with 36 pears.

A controller notes that many of the pears inspected in question 3a were worm eaten. Paul thinks that the number of worm eaten pears is not that bad and he claims that 5% of his pears are worm eaten. The inspector, on the other hand, believes that more than 5% of Paul's pears are worm eaten.

The inspector and Paul decide to subject this disagreement to a test procedure. To this end, they take a sample of 100 randomly selected pears from Paul's harvest and count the number of worm eaten pears in this sample. They thereby agree a significance level of $\alpha = 5\%$.

2pt c Formulate the null hypothesis and the alternative hypothesis for this testing procedure.

The outcome of the sample is that 8 of the 100 pears are worm eaten.

4pt d Compute the probability that if Paul is right, there are exactly 8 worm eaten pears in the sample.

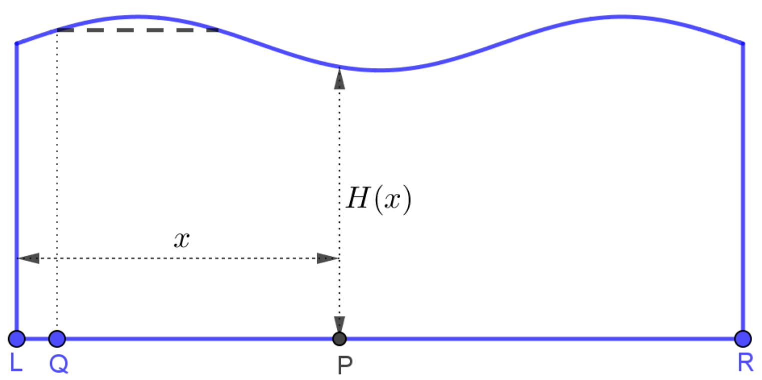
4pt e On the basis of your answer to item 3d, can you formulate a conclusion for this test procedure? If so, formulate and motivate this conclusion as accurately as possible. If not, what probability would you compute in order to be able to formulate a conclusion?

Extra items on the last page of this example exam.

Question 4 – A wavy roof

Source: CCVX entrance exam wiskunde A April 2018

The figure below shows a cross section of a sports hall with a wavy roof. The lowest point of this roof is 10 m above the floor, the highest point is 12 m above the floor. The width of the sports hall (that is the distance *LR*) is 27 meters.



The height of the roof above a certain point *P* on the floor is a function of the distance x of this point to point *L*. To this function fits a formula of the form

$$H(x) = A + B\sin(Cx)$$

with H and x in meters.

3pt a Show that the height of the roof above point *L* is equal to *A* and compute this height, that is, the value of A.

Furthermore, it is given that the height of the roof above point *R* also equals *A*.

4pt b Now, compute values of *B* and *C* which match the above description.

Point *Q* is on the floor between point *L* and point *R* at a distance of 1.5 meter from point *L*. Straight above point *Q* a cable is attached to the roof, which runs horizontally in the plane of the cross section. The other end of this cable is also attached to the roof, see the striped line in the figure above.

3pt c Compute the length of this cable algebraically.

Question 5 – Two growth models

New

The company Fooglebook has developed a new app. Fooglebook has developed models for the number of users of this app and for the revenue that is generated by this app for the company. The first model is based on 5000 users at the time of the introduction of the app and an increase of this number by 10% per month.

5pt a Compute the time in which the number of users doubles according to this model.

The monthly revenue that the app generates for the company according to this first model is given by the formula $I(n) = 10\,000 + 400n$.
In this formula, $I(n)$ indicates the revenue in euros in month n with $n = 1$ in the month in which the app is introduced.

5pt b Without computing the revenue for each month separately, compute the total revenue that the app generates according to this model in the first two years after its introduction. (1 year = 12 months).

According to this first model, the number of users will increase more and more quickly and moreover it will eventually rise above every limit. Because this is not realistic, another model is developed in which the number of users first rises, but then levels to a limit value. The number of users according to this second model is given by the formula $A(t) = 5000 \cdot (100 - 99 \cdot \mathrm{e}^{-0.5t})$.
In this formula, $A(t)$ is the number of users and t is the time in months with $t = 0$ at the time of the introduction of the app.

3pt c Compute the limit value of the number of users according to the second model.

4pt d Compute the rate of growth of the number of users according to the second model at $t = 12$. (rate of growth = derivative of the growth function)

The formula $A = 5000 \cdot (100 - 99 \cdot \mathrm{e}^{-0.5t})$ can be transformed into a formula with which the time can be computed when the number of users is given.

4pt e Compute this formula and use this formula to determine the time at which, according to the second model, the app has 490 000 users.

Question 6 – Three dice

Source: CCVX entrance exam wiskunde A April 2018

Jeroen is on the King's Day free market with the following game:

- a player pays a bet to Jeroen and then throws three dice
- if the player throws three fives, Jeroen pays 50 euros to the player
- at two fives, Jeroen pays 8 euros to the player
- with one five Jeroen pays 2 euros to the player
- if no five is thrown, the player loses his bet.

3pt **a** Show that the probability of the player being paid 8 euro's is equal to $\frac{15}{216}$.

6pt **b** Make a probability distribution of the pay-out per game and compute how big the player's bet should minimally be to achieve that Jeroen is expected to make a profit with this game.

Extra items for question 3

The controller also wonders whether the mean weight of Paul's pears is 190 grams indeed. To test this, he determines the mean of the weights of 25 randomly selected pears. Thereby, he assumes that the standard deviation of 7 grams is correct and he takes a significance level of $\alpha = 0.05$.

2pt **f** Formulate the null hypothesis and the alternative hypothesis for this testing procedure.

The mean weight of the pears in the sample turns out to be 187.5 grams.

5pt **g** What is the result of the testing procedure with this outcome?

End of the exam.

Is your name on all answer sheets?

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Differentiation

Rule	function	derivative function
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left(g(x)\right)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\left(g(x)\right) \cdot g'(x)$ or $\frac{dk}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$

Logarithms

Rule	conditions
${}^{g}\log a + {}^{g}\log b = {}^{g}\log ab$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^{g}\log a - {}^{g}\log b = {}^{g}\log \frac{a}{b}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^{g}\log a^p = p \cdot {}^{g}\log a$	$g > 0, \quad g \neq 1, \quad a > 0$
${}^{g}\log a = \frac{{}^{p}\log a}{{}^{p}\log g}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad p > 0, \quad p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \frac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* =number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If X and Y are any random variables, then: $E(X+Y)=E(X)+E(Y)$
If furthermore X and Y are independent, then: $\sigma(X+Y)=\sqrt{\sigma^2(X)+\sigma^2(Y)}$

$\sqrt{n}$-law:
For n independent repetitions of the same experiment where the result of each experiment is a random variable X, the sum of the results is a random variable S and the mean of the results is a random variable $\overline{X}$.

$E(S)=n\cdot E(X)$ $\qquad$ $\sigma(S)=\sqrt{n}\cdot\sigma(X)$

$E(\overline{X})=E(X)$ $\qquad$ $\sigma(\overline{X})=\dfrac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If X has a binomial distribution with parameters n (number of experiments) and p (probability of success at each experiment), then

$$P(X=k)=\binom{n}{k}\cdot p^k\cdot(1-p)^{n-k} \quad \text{with} \quad k=0,1,2,\ldots,n$$

Expected value: $E(X)=np$ $\qquad$ Standard deviation: $\sigma(X)=\sqrt{n\cdot p\cdot(1-p)}$

n and p are the parameters of the binomial distribution

Normal Distribution

If X is a normally distributed random variable with mean μ and standard deviation σ, then

$Z=\frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X<g)=P\left(Z<\frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic T is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are::

α	left sided	right sided	two sided
0.05	$g=\mu_T-1.645\sigma_T$	$g=\mu_T+1.645\sigma_T$	$g_l=\mu_T-1.96\sigma_T$
			$g_r=\mu_T+1.96\sigma_T$
0.01	$g=\mu_T-2.33\sigma_T$	$g=\mu_T+2.33\sigma_T$	$g_l=\mu_T-2.58\sigma_T$
			$g_r=\mu_T+2.58\sigma_T$