Elaborations Example Exam 1 Wiskunde A 2018

Question 1a – 4 points

$3x^2 - 7x + 2 = 0$ yields $D = 7^2 - 4 \cdot 3 \cdot 2 = 49 - 24 = 25$

Solutions: $x = \frac{7-5}{2 \cdot 3} = \frac{2}{6} = \frac{1}{3}$ and : $x = \frac{7+5}{2 \cdot 3} = \frac{12}{6} = 2$

Question 1b – 4 points

$2x^4 - 4x^8 = 0 \Leftrightarrow 2x^4(1 - 2x^4) = 0 \; of \; x^4(2 - 4x^4) = 0 \Leftrightarrow x^4 = 0 \; of \; 1 - 2x^4 = 0$

$x^4 = 0 \Leftrightarrow x = 0$; $x^4 = \frac{1}{2} \Leftrightarrow x = \pm\left(\frac{1}{2}\right)^{\frac{1}{4}} = \pm\sqrt[4]{\frac{1}{2}} \approx \pm 0{,}841$

Question 1c – 4 points

$2 \cdot 4^x - 4 \cdot 8^x = 0 \Leftrightarrow 2 \cdot 4^x = 4 \cdot 8^x \Leftrightarrow \frac{8^x}{4^x} = \frac{2}{4} \Leftrightarrow 2^x = \frac{1}{2} \Leftrightarrow x = -1$

Alternative: $2^1 \cdot 2^{2x} = 2^2 \cdot 2^{3x} \Leftrightarrow 2^{1+2x} = 2^{2+3x} \Leftrightarrow 1 + 2x = 2 + 3x \Leftrightarrow x = -1$

Question 1d – 4 points

$2 \cdot 5^x - 30 = 0 \Leftrightarrow 2 \cdot 5^x = 30 \Leftrightarrow 5^x = 15 \Leftrightarrow x = {}^5\log(15) \approx 1{,}683$

Question 1e – 4 points

$\sqrt{4x-7} = \frac{1}{2}x + 1 \Rightarrow 4x - 7 = \left(\frac{1}{2}x + 1\right)^2 \Leftrightarrow 4x - 7 = \frac{1}{4}x^2 + x + 1 \Leftrightarrow \frac{1}{4}x^2 - 3x + 8 = 0$

$$\Leftrightarrow x = \frac{3 + \sqrt{(-3)^2 - 4 \cdot \frac{1}{4} \cdot 8}}{2 \cdot \frac{1}{4}} = \frac{3+1}{\frac{1}{2}} = 8 \vee x = x = \frac{3 - \sqrt{(-3)^2 - 4 \cdot \frac{1}{4} \cdot 8}}{2 \cdot \frac{1}{4}} = \frac{3-1}{\frac{1}{2}} = 4$$

Alternative $x^2 - 12x + 32 = 0 \Leftrightarrow (x-4)(x-8) = 0.$

Intersections (4,3) and (8,5)

Extra item 1f – 4 points

$f'(x) = \frac{4}{2\sqrt{4x-7}} = \frac{2}{\sqrt{4x-7}}$, so $f'(2) = \frac{2}{\sqrt{8-7}} = \frac{2}{1} = 2$

Question 2a – 2 points

$Revenue = q \cdot p(q) = 14q - 0.05q^2$

$Total\ cost = 250 + 2q$

$Profit = 14q - 0.05q^2 - (250 + 2q) = 14q - 0,05q^2 - 250 - 2q = -0.05q^2 + 12q - 250$

Question 2b – 3 points

$w'(q) = -0.1q + 12$

$w'(q) = 0 \Leftrightarrow -0.1q + 12 = 0 \Leftrightarrow q = 120$

Question 2c – 6 points

$$R'(q) = \frac{(-10q + 1200)(250 + 2q) - (-5q^2 + 1200q - 25\,000) \cdot 2}{(250 + 2q)^2}$$

Expanding brackets in the numerator yields

$-2500q - 20q^2 + 300\,000 + 2400q + 10q^2 - 2400q + 50\,000 = -10q^2 - 2500q + 350\,000$

$R'(q) = 0 \Leftrightarrow q^2 + 250q - 35\,000 = 0 \Leftrightarrow (q - 100)(q + 350) = 0 \Leftrightarrow q = 100\ of\ q = -350$

You may also use the quadratic formula.

Only $q = 100$ suffices.

Question 3a – 4 points

For each pear, the probability that it weighs mora than 190 grams is $\frac{1}{2}$

The probability that all 10 pears weigh more than 190 grams is therefore $\left(\frac{1}{2}\right)^{10} = \frac{1}{1024} \approx 0.000977$

Question 3b – 5 points

$\mu_{total} = \mu_{pears} + \mu_{box} = 36 \cdot 190 + 500 = 7340$ gram

$\sigma_{total}^2 = \sigma_{pears}^2 + \sigma_{box}^2 = 36 \cdot 7^2 + 20^2 = 2164; \quad \sigma_{total} = \sqrt{2164} \approx 46{,}52$ grams.

Question 3c – 2 points

$H_0\colon\ p = 0.05;\ \ H_1\colon\ p > 0.05$

Question 3d – 4 points

$\binom{100}{8} \cdot 0.05^8 \cdot 0.95^{92} \approx 0.06474$

Question 3e – 4 points

You should compute the p-value $P(X \geq 8)$, but since this probability exceeds the probability that you computed in item 3d and since this probability in turn exceeds α, you know that $P(X \geq 8) > \alpha$, so H_0 is not rejected. There is not sufficient evidence to support the statement that more than 5% of the pears is worm eaten.

Extra item 3f – 2 points

$H_0\colon\ \mu = 190;\ \ H_1\colon\ \mu \neq 190$

Extra item 3g – 5 points

The mean weight of the 25 pears is normally distributed with $\mu_G = 190$ grams
and $\sigma_G = \frac{7}{\sqrt{25}} = 1{,}4$ grams.
The limit of the left hand side rejection area is therefore
$g_l = \mu_G - 1.96\sigma_G = 190 - 1.96 \cdot 1{,}4 = 187.26$ grams.
Since the mean weight of the sample is 187.5 grams, this means that H_0 is not rejected. There is not sufficient evidence to support the statement that the mean weight is not 190 grams.

Question 4a – 3 points

The height of the roof above *L* is given by $H(0) = A + B\sin(0) = A + B \cdot 0 = A$

A is the equilibrium..

The maximal height is 12 m, the minimal height is 10m. The equilibrium *A* is therefore 11 m.

Question 4b – 4 points

The amplitude *B* is the difference between the maximal height and the equilibrium, that is 1 m.

$27\ m = 1.5$ period, so the period is 18 m.

This yields $C = \frac{2\pi}{periode} = \frac{2\pi}{18} = \frac{1}{9}\pi\ (\approx 0.349)$

Question 4c – 3 points

Above *L* (at $x = 0$) the height is equal to the amplitude of 11 m.

Half a period, that is 9 m, to the right, the height is 11 m again..

Because of the symmetry, the height at 1.5 m to the left of this point is equal to the height 1.5 m to the right of point *L*, that is 7.5 m to the right of point *L* or 6 m to the right of the left end of the cable.

The answer can also be found by solving the equation $H(x) = H(1.5) = 11.5$ *and checking which of the solutions fulfil the description. However, this method is not a part of the wiskunde A program.*

Question 5a – 5 points

The growth factor over 1 month is $g = 1 + \frac{10}{100} = 1.1$.

For the doubling time *T* we have $g^T = 2$

This yields $1.1^T = 2 \Leftrightarrow T = {}^{1.1}\log(2) \approx 7.27$ months

This answer can also be found by solving the equation $5000 \cdot 1.1^t = 10\,000$.

Question 5b – 5 points

The revenues per month are an arithmetic sequence with $I(1) = 10\,000 + 400 = 10\,400$ and $I(24) = 10\,000 + 400 \cdot 24 = 19\,600$

The sum of the first 24 terms of this sequence is:

$\frac{1}{2} \cdot 24 \cdot \left(I(0) + I(24)\right) = \frac{1}{2} \cdot 24 \cdot (10\,400 + 19\,600) = 360\,000$

Question 5c – 3 points

Eventually, $\mathrm{e}^{-0.5t}$ becomes practically 0.

The limit value is therefore $5000 \cdot (100 - 0) = 500\,000$.

Question 5d – 4 points

$A(t) = 5000 \cdot (100 - 99 \cdot \mathrm{e}^{-0.5t}) = 500\,000 - 495\,000 \cdot \mathrm{e}^{-0.5t}$

yields $A'(t) = -495\,000 \cdot (-0.5 \cdot \mathrm{e}^{-0.5t}) = 247\,500 \cdot \mathrm{e}^{-0.5t}$

This yields $A'(12) = 247\,500 \cdot \mathrm{e}^{-6} \approx 613.49$ euros per month

Question 5e – 4 points

$A = 5000 \cdot (100 - 99 \cdot \mathrm{e}^{-0.5t}) \Leftrightarrow 100 - 99 \cdot \mathrm{e}^{-0.5t} = \frac{1}{5000}A \Leftrightarrow -99 \cdot \mathrm{e}^{-0.5t} = \frac{1}{5000}A - 100$

$\Leftrightarrow \mathrm{e}^{-0.5t} = \frac{100}{99} - \frac{1}{495\,000}A \Leftrightarrow -0.5t = \ln\left(\frac{100}{99} - \frac{1}{495\,000}A\right) \Leftrightarrow t = -2\ln\left(\frac{100}{99} - \frac{1}{495\,000}A\right)$

$A = 495\,000$ then yields $t = -2\ln\left(\frac{100}{99} - 1\right) \approx 9.19$ months

Question 6a – 3 points

$P(5,5, no\ 5) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{5}{216}$

There are 3 combinations with two times a 5 and on ti,e another result, so the probability in this question is indeed $\frac{15}{216}$

Question 6b – 6 points

The probability of three times a 5 is $\left(\frac{1}{6}\right)^3 = \frac{1}{216}$, the probability of no 5 is $\left(\frac{5}{6}\right)^3 = \frac{125}{216}$

The probability of one 5 is $\frac{75}{216}\left(= 1 - \left(\frac{125}{216} + \frac{15}{216} + \frac{1}{216}\right) = 3 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^2\right)$

The probability distribution is therefore

$P(U = 0) = \frac{125}{216}$; $P(U = 2) = \frac{75}{216}$; $P(U = 8) = \frac{15}{216}$; $P(U = 50) = \frac{1}{216}$

This yields $E(U) = 0 + 2 \cdot \frac{75}{216} + 8 \cdot \frac{15}{216} + 50 \cdot \frac{1}{216} = \frac{320}{216} \approx 1.481$

Therefore, the bet must be at least € 1,49.