CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Answers & brief elaborations Wiskunde A 19 December 2018

1a $9x^3+25x=30x^2 \Leftrightarrow 9x^3-30x^2+25x=0 \Leftrightarrow x(9x^2-30x+25)=0$

$9x^2-30x+25=0 \Leftrightarrow (3x-5)^2=0 \Leftrightarrow 3x=5 \Leftrightarrow x=\frac{5}{3}$

of / *or* $D=30^2-4\cdot 9\cdot 25=900-900=0 \Rightarrow x=\frac{-30}{2\cdot 9}=\frac{5}{3}$

Solutions: $x=0$; $x=\frac{5}{3}$

1b $5\cdot 4^x=2\cdot 5^x \Leftrightarrow \dfrac{4^x}{5^x}=\dfrac{2}{5} \Leftrightarrow \left(\frac{4}{5}\right)^x=\frac{2}{5} \Leftrightarrow x={}^{0,8}\log(0{,}4)\approx 4{,}106$

1c $f'(x)=\dfrac{8}{2\cdot\sqrt{8x-12}} \Rightarrow f'(2)=\dfrac{8}{2\cdot 2}=2$

This is exactly the slope of line l.

1d $\sqrt{8x-12}=2x-6 \Rightarrow 8x-12=(2x-6)^2 \Leftrightarrow 8x-12=4x^2-24x+36$

$\Leftrightarrow 4x^2-32x+48=0 \Leftrightarrow x^2-8x+12=0 \Leftrightarrow x=2 \vee x=6$

$x=6$ is the only solution of the original equation.

Intersection: (6,6)

2a $\dfrac{2\pi}{\pi/13}=2\pi\cdot\dfrac{13}{\pi}=26$ weeks

2b The maximal value of the sine is 1, therefore $B_{max}=5400+200\cdot 1=5400$

2c $5000=5200-200$ is the minimal value of *B*.

B has a starting point at $t=8\frac{1}{2}$. The next minimum is after $\frac{3}{4}$ period,

that is at $t=8\frac{1}{2}+\frac{3}{4}\cdot 26=\ 8\frac{1}{2}+19\frac{1}{2}=28$

The other minimum is $t=8\frac{1}{2}-\frac{1}{4}\cdot 26=2\ \ (=28-26)$

2c *Alternative*

$5200+200\sin\left(\dfrac{\pi}{13}\left(t-8\tfrac{1}{2}\right)\right)=5000 \Leftrightarrow \sin\left(\dfrac{\pi}{13}\left(t-8\tfrac{1}{2}\right)\right)=-1$

$\sin\left(-\tfrac{1}{2}\pi\right)=-1 \Rightarrow \dfrac{\pi}{13}\left(t-8\tfrac{1}{2}\right)=-\tfrac{1}{2}\pi \Leftrightarrow t-8\tfrac{1}{2}=-6\tfrac{1}{2} \Leftrightarrow t=2$

$\sin\left(1\tfrac{1}{2}\pi\right)=-1 \Rightarrow \dfrac{\pi}{13}\left(t-8\tfrac{1}{2}\right)=1\tfrac{1}{2}\pi \Leftrightarrow t-8\tfrac{1}{2}=19\tfrac{1}{2} \Leftrightarrow t=28\ \ (=2+26)$

2d On average, there are 5200 births per week, so there are $5200\times 52=270\,400$ births in a year

3a $\frac{13t}{t^2+4} = 1.25 \Leftrightarrow 13t = 1.25t^2 + 5 \Leftrightarrow 1.25t^2 - 13t + 5 = 0$

$\Leftrightarrow t = \frac{13 \pm \sqrt{13^2 - 4 \cdot 1.25 \cdot 5}}{2 \cdot 1.25} = \frac{13 \pm \sqrt{144}}{2.5} \Leftrightarrow t = \frac{13+12}{2.5} = 10 \vee t = \frac{13-12}{2.5} = 0.4$

The drug is effective during $10 - 0.4 = 9.6$ hours, that is 576 minutes.

3b $C_1'(t) = \frac{13(t^2+4) - 13t \cdot 2t}{(t^2+4)^2} = \frac{52 - 13t^2}{(t^2+4)^2}$

$C_1'(t) = 0 \Leftrightarrow 52 - 13t^2 = 0 \Leftrightarrow t^2 = 4 \Leftrightarrow t = 2$ (N.B.: $t \geq 0$)

$C_1(2) = \frac{13 \cdot 2}{4+4} = \frac{26}{8} = 3.25 \text{ mg/l}$

3c $C_1(3) = \frac{13\cdot3}{9+4} = 3$; $C_2(3) = 4.5 \cdot 3\mathrm{e}^{-1,5} = 3.0123$

$\frac{C_2(3) - C_1(3)}{C_1(3)} \times 100\% = \frac{0.0123}{3} \times 100\% = 0.41\%$

3d $C_2'(t) = 4.5\mathrm{e}^{-0.5t} + 4.5t \cdot (-0.5\mathrm{e}^{-0.5t})$

$C_2'(2) = 4.5 \cdot \mathrm{e}^{-1} + 4.5 \cdot 2 \cdot (-0.5\mathrm{e}^{-1}) = 4.5\mathrm{e}^{-1} - 4.5\mathrm{e}^{-1} = 0$

4a $P(X > 7) = P(X = 8) + P(X = 9) + P(X = 10)$

$= \binom{10}{8} \cdot 0{,}6^8 \cdot 0{,}4^2 + 10 \cdot 0{,}6^9 \cdot 0{,}4 + 0{,}6^{10} = 0.1673$

4b Code: G = valid; N = not valid

$P(X = 2) = P(NG) = \frac{3}{6} \cdot \frac{3}{5} = \frac{3}{10}$; $P(X = 3) = P(NNG) = \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{3}{4} = \frac{3}{20}$

4c $P(X = 1) = P(NG) = \frac{3}{6} = 0.50$; $P(X = 2) = 0.30$; $P(X = 3) = 0.15$

$P(X = 4) = P(NNNG) = \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{1}{4} \cdot \frac{3}{3} = \frac{1}{20}$ $\big(= 1 - (0{,}50 + 0{,}30 + 0{,}15)\big)$

4d $E(X) = 1 \cdot 0.50 + 2 \cdot 0.30 + 3 \cdot 0.15 + 4 \cdot 0.05 = 1.75$

5a $1{,}01^{52} = 1{,}6777 \Rightarrow 67{,}77\%$

5b The numbers of toys sold per week are a geometric sequence with $r = 1.01$

$S(1) = 10\,000 \cdot 1{,}01^{0} = 10\,000$; $S(53) = 10\,000 \cdot 1{,}01^{52} = 16\,776.89$

$$\text{Sum} = \frac{S(53) - S(1)}{1.01 - 1} = \frac{16\,776.89 - 10\,000}{0.01} = 677\,689$$

5c $\dfrac{100\,000}{2 + 8\mathrm{e}^{-0.014t}} = 25\,000 \Leftrightarrow 2 + 8\mathrm{e}^{-0.014t} = 4 \Leftrightarrow \mathrm{e}^{-0.014t} = 0.25 \Leftrightarrow -0.014t = \ln(0.25)$

$$t = \frac{\ln(0.25)}{-0.014} = 99.0\ weeks$$

5d In the long run, $\mathrm{e}^{-0.014t}$ becomes practically 0, so

$$S(t) \to \frac{100\,000}{2 + 0} = 50\,000 \quad (t \to \infty)$$

6a According to the rules of thumb, 68% of the scores is in between $\mu - \sigma$ and $\mu + \sigma$

The symmetry of the normal distribution then yields:

$(100\% - 68\%) \cdot \frac{1}{2} = 16\%$ exceeds $\mu + \sigma$

According to the rules of thumb, 95% of the scores is in between $\mu - 2\sigma$ and $\mu + 2\sigma$

The symmetry of the normal distribution then yields

$(100\% - 95\%) \cdot \frac{1}{2} = 2.5\%$ exceeds $\mu + 2\sigma$

$16\% - 2.5\% = 13.5\%$ of the scores is "Good"; $13.5\% \times 3000 = 405$

6b $H_0\colon\ \mu = 550$; $H_1\colon\ \mu \neq 550$

6c The test statistic T is normally distributed with $\mu_T = 550$ en $\sigma_T = \frac{90}{\sqrt{100}} = 9$

6d $0.029 > \frac{1}{2}\alpha = 0.025$

The null hypotheses is not rejected. There is not enough evidence to state that the mean score is different from 550.

