CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE
Worked Solutions Wiskunde A 19 April 2019

1a $2x^2 - 3x^3 = x \Leftrightarrow 3x^3 - 2x^2 + x = 0 \Leftrightarrow x(3x^2 - 2x + 1) = 0$
$3x^2 - 2x + 1 = 0$ yields $D = b^2 - 4ac = 4 - 4 \cdot 3 \cdot 1 = -8$
$D < 0$, so $3x^2 - 2x + 1 = 0$ has no solution.
The only solution is therefore $x = 0$.

1b $2x - 3y = 5 \Leftrightarrow 3y = 2x - 5 \Leftrightarrow y = \frac{2}{3}x - \frac{5}{3}$
m is therefore the line through the origin with slope $\frac{2}{3}$, that is the line $y = \frac{2}{3}x$
Intersection with *l*: $3x - 2 = \frac{2}{3}x \Leftrightarrow \frac{7}{3}x = 2 \Leftrightarrow x = \frac{6}{7}$ and $y = \frac{2}{3} \cdot \frac{6}{7} = \frac{4}{7}$

1c $f'(x) = \dfrac{6(x^2+9) - 6x \cdot 2x}{(x^2+9)^2} = \dfrac{54 - 6x^2}{(x^2+9)^2}$
$f'(x) = 0 \Leftrightarrow 54 - 6x^2 = 0 \Leftrightarrow x^2 = 9 \Leftrightarrow x = 3 \vee x = -3$
Minimum: $f(-3) = \frac{-18}{9+9} = -1$; maximum: $f(3) = \frac{18}{9+9} = 1$

1d $f(x) = g(x) \Leftrightarrow \dfrac{6x}{x^2+9} = \dfrac{3}{2x+3} \Leftrightarrow 6x(2x+3) = 3(x^2+9) \Leftrightarrow 12x^2 + 18x = 3x^2 + 27$
$\Leftrightarrow 9x^2 + 18x - 27 = 0 \Leftrightarrow x^2 + 2x - 3 = 0 \Leftrightarrow (x-1)(x+3) = 0 \Leftrightarrow x = 1 \vee x = -3$
$f(1) = g(1) = \frac{3}{5}$; $f(-3) = g(-3) = -1$, so intersections $\left(1, \frac{3}{5}\right)$ and $(-3, -1)$.

2a In the graph we can see: $T(maximal) \approx 37.5$℃ and $T(minimal) \approx 36.5$℃
This yields $A = \frac{37.5+36.5}{2} = 37.0$℃ and $B = 37.5 - A = 0.5$℃

2b 0.262 must be equal to $\frac{2\pi}{period}$.
From the graph (or the context) we can conclude: $period = 24$ (h)
This yields $\frac{2\pi}{period} = \frac{2\pi}{24} \approx 0.2618$

2c The temperature is maximal when the sine is maximal, that is when
$0.262(t + 1.45) = \frac{1}{2}\pi$. This yields $t + 1.45 = \frac{\frac{1}{2}\pi}{0.262} = 6.00$.
Also: The sine is maximal at $\frac{1}{4}\text{period} = 6\text{ h}$ after passing the equilibrium at $t = -1.45$
In both ways, we get $t = 4.55$
That is 4 h and $0.55 \cdot 60 = 33$ minutes after 12:00 h, that is at 16:33 h.

3a At $Q = 12$ we have $Price(12) = 10 - \sqrt{3 \cdot 12} = 10 - \sqrt{36} = 10 - 6 = 4$,
so the revenue in euros is $12\,000 \cdot 4 = 48\,000$.
$Profit(12) = 7 \cdot 12 - \sqrt{3 \cdot 12^3} - 6 = 84 - \sqrt{5184} - 6 = 84 - 72 - 6 = 6$,
so the profit in euros is 6000

3b The revenue (in thousands of euros) as a function of *Q* is given by
$R(Q) = P(Q) \cdot Q = \left(10 - \sqrt{3Q}\right) \cdot Q = 10Q - \sqrt{3Q} \cdot \sqrt{Q^2} = 10Q - \sqrt{3Q^3}$
The cost (in thousands of euros) is therefore given by
$C(Q) = R(Q) - W(Q) = 10Q - \sqrt{3Q^3} - \left(7Q - \sqrt{3Q^3} - 6\right) = 3Q + 6$

3c The chain rule yields $W'(Q) = 7 - \frac{1}{2\sqrt{3Q^3}} \cdot 3 \cdot 3Q^2$
With $W(Q) = 7Q - \sqrt{3} \cdot Q^{\frac{3}{2}} - 6$ we get $W'(Q) = 7 - \frac{3}{2}\sqrt{3} \cdot \sqrt{Q}$
$W'(12) = 7 - 9 = -2$
$W'(12) < 0$, so the profit decreases if *Q* increases.
To increase the profit, the production must be reduced!

3d $W'(Q) = 3 \cdot \mathrm{e}^{-0.4Q} + 3Q \cdot \mathrm{e}^{-0.4Q} \cdot -0.4 = (3 - 1.2Q) \cdot \mathrm{e}^{-0.4Q}$,
dus $W'(2.5) = (3 - 1.2 \cdot 2.5)\mathrm{e}^{-0{,}4 \cdot 2{,}5} = (3 - 3) \cdot \mathrm{e}^{-1} = 0$

4a $E(Y) = 1 \cdot P(Y = 1) + 2 \cdot P(Y = 2) + 3 \cdot P(X = 3) + 4 \cdot P(Y = 4)$
$= 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4} + 4 \cdot \frac{1}{4} = \frac{10}{4} = 2\frac{1}{2}$

4b $\sigma(S) = \sqrt{\sigma^2(X) + \sigma^2(Y)} = \sqrt{\frac{35}{12} + \frac{5}{4}} = \sqrt{\frac{35}{12} + \frac{15}{12}} = \sqrt{\frac{50}{12}} = \sqrt{\frac{25}{6}} \approx 2.04$

4c $E(S) = E(X) + E(Y) = 3\frac{1}{2} + 2\frac{1}{2} = 6$
$P(S = 6) =$
$P(X = 2 \wedge Y = 4) + P(X = 3 \wedge Y = 3) + P(X = 4 \wedge Y = 2) + P(X = 5 \wedge Y = 1)$
$= 4 \cdot \frac{1}{6} \cdot \frac{1}{4} = \frac{1}{6}$

4d $P(outcome \neq 6 \; at \; one \; throw) = \frac{5}{6}$
$P(outcome \neq 6 \; at \; 8 \; out \; of \; 10 \; throws) = \binom{10}{8} \cdot \left(\frac{5}{6}\right)^8 \cdot \left(\frac{1}{6}\right)^2 \approx 0.29071$
$P(outcome \neq 6 \; at \; 9 \; out \; of \; 10 \; throws) = \binom{10}{9} \cdot \left(\frac{5}{6}\right)^9 \cdot \frac{1}{6} \approx 0.32301$
$P(outcome \neq 6 \; at \; 10 \; out \; of \; 10 \; throws) = \left(\frac{5}{6}\right)^{10} \approx 0.16151$
$P(outcome \neq 6 \; at \; 7 \; or \; less \; out \; of \; 10 \; throws)$
$= 1 - (0.29071 + 0.32301 + 0.16151) = 0.22477$

5a $200 = 203 - 3 = \mu_A - \frac{1}{2}\sigma_A$

PA is therefore $0.191 + 0.191 + 0.150 + 0.136 + 0.023 = 0.191 + 0.500 = 0.691$

5b $200 = 207 - 7 = \mu_B - \frac{7}{12}\sigma_B$

The boundary of the area that represents *PB* is therefore left of $\mu - \frac{1}{2}\sigma = 201$.

Hence, PB > PA.

5c H_0: $\mu = 205$; H_1: $\mu \neq 205$

5d The test statistic *T* is normally distributed with $\mu_T = 205$ and $\sigma_T = \frac{10}{\sqrt{16}} = 2.5$.

The boundary of the rejection area of a two sided test with $\alpha = 5\%$ is

$g_l = \mu_T - 1.96\sigma_T = 205 - 1.96 \cdot 2.5 = 200.1$

The observed sample result is larger than this boundary, so H_0 is not rejected.

There is not enough evidence to reject the claim of lumber store C.

6a The doubling time *T* is found by solving

$1.5^T = 2 \Leftrightarrow T = {}^{1.5}\log(2) \approx 1.7095$ hour ≈ 1 hour and 43 minutes

6b $W_E(10) = 600 \cdot 1.5^{10} \approx 34599$

$W_B(10) = 250 \cdot (700 - 1527\mathrm{e}^{-0.1\cdot 10}) = 250 \cdot (700 - 1527\mathrm{e}^{-1}) \approx 34562$

6c The equation to solve is: $250 \cdot (700 - 1527\mathrm{e}^{-0.1t}) = 0.17 \cdot 10^6$

Dividing l.h.s and r.h.s. by 250 yields $700 - 1527\mathrm{e}^{-0.1t} = 680,$ so

$1527\mathrm{e}^{-0.1t} = 20 \Leftrightarrow \mathrm{e}^{-0.1t} = \frac{20}{1527} \Leftrightarrow -0.1t = \ln\left(\frac{20}{1527}\right) \Leftrightarrow t = -10 \cdot \ln\left(\frac{20}{1527}\right) \approx 43.35$

6d When *t* becomes large, $\mathrm{e}^{-0.1t}$ becomes almost 0

Hence, the weight becomes almost $250 \cdot 700 = 175\,000$ microgram.