

CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Example Exam Wiskunde B

Date: Autumn 2018
Time: 3 Hours
Questions: 6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in reduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put in your bag.

Points that can be scored for each question:						
Question	1	2	3	4	5	6
a	4	3	6	5	4	4
b	4	6	4	6	2	6
c	4	5		6	4	5
d		6			5	
Total	12	20	10	17	15	15
Grade = $\frac{\text{total points scored}}{10} + 1$						
You will pass the exam if your grade is at least 5.5 .						

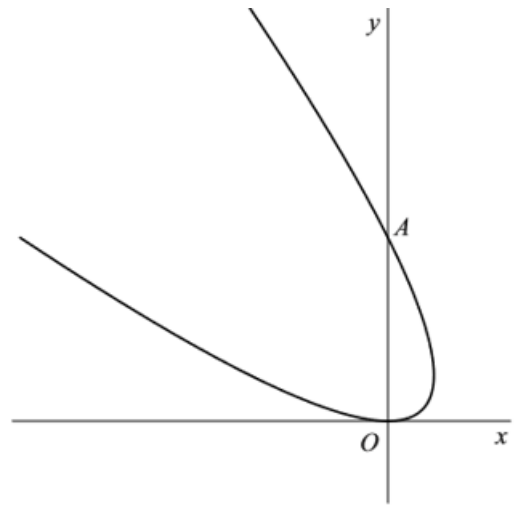
Question 1

Source: Centraal Examen vwo 2018 tijdvak 1 , with question c added

The movement of a point P is given by the following parametric equations:

$$\begin{cases} x(t) = 1 - t^2 \\ y(t) = (1 + t)^2 \end{cases}$$

In the figure on the right the path of P is shown.



The path of P intersects the y -axis in the origin O and in point A (see the figure)

- 4pt a Compute exactly the velocity of point P when it passes point A .
- 4pt b Show that for each value of t , point P is on the curve with equation $(x + y)^2 = 4y$.

The straight line ℓ intersects the path of P perpendicularly in point A .

- 4pt c Use an exact computation to find an equation for line ℓ .

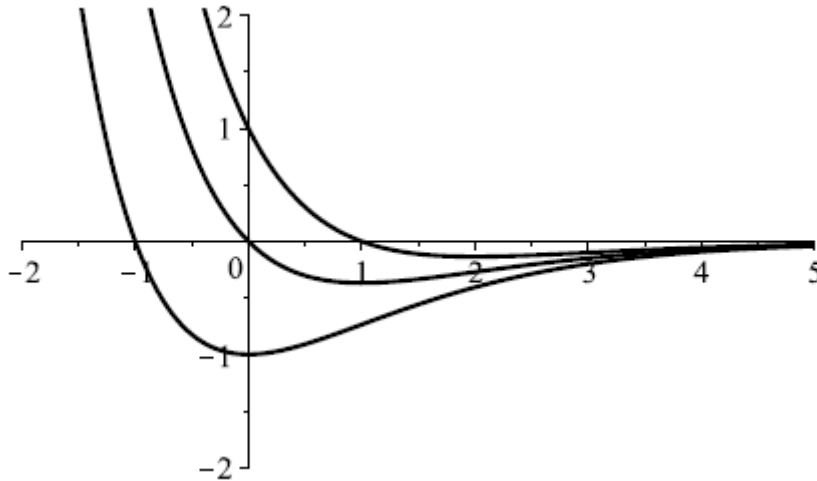
Extra item on the last page of this example exam.

Question 2

Source: CCVX entrance exam wiskunde B April 2018

The family of functions f_p is given by $f_p(x) = (p - x)e^{-x}$.

Below the graphs are shown of the functions f_{-1} , f_0 and f_1



3pt a Show by means of an exact computation that $F(x) = (x + 1)e^{-x}$ is an antiderivative of f_0 .

6pt b Compute exactly the area of the region enclosed by the x-axis, the y-axis and the graph of f_1 .

For $p > 0$, V_p is the region enclosed by the y-axis, the graph of f_p , the graph of f_{-p} and the vertical line $x = 1$.

5pt c Compute exactly the value of p for which the area of region V_p equals 2.

For each real number p , the graph of f_p has one point of inflection, B_p .

The points of inflection B_p are all one a curve.

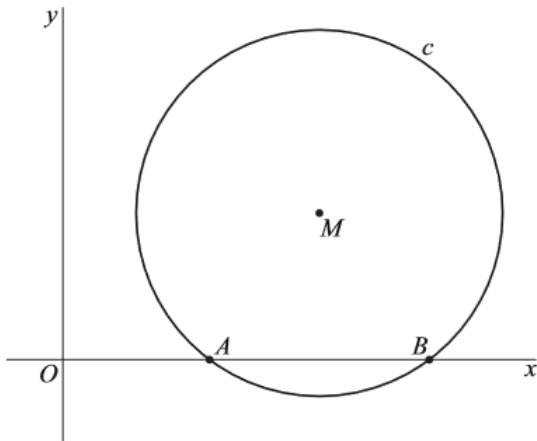
6pt d Use an exact computation to find an equation for the curve that contains all points B_p .

Question 3

Source: Centraal Examen vwo 2018 tijdvak 1 (adjusted)

Given is the circle c with centre $M(14,8)$ and radius 10. This circle intersects the x -axis in the points A and B . See *figuur 1*.

figuur 1

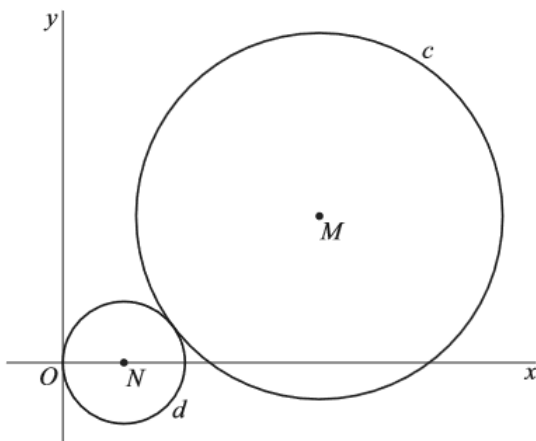


Circle c^* is the circle that passes through points A , B and M .

6pt a Compute exactly the radius of circle c^* .

The circle d with centre N touches the y -axis in the origin O and touches circle c as indicated in *figuur 2*.

figuur 2



4pt b Compute exactly the radius of circle d .

Question 4

Source: CCVX entrance exam wiskunde B April 2018

The functions f and g are given by

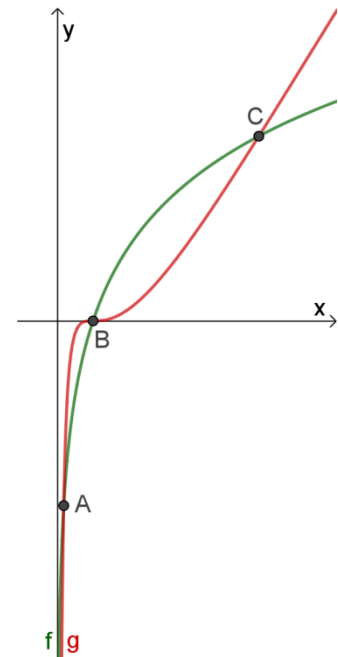
$$f(x) = 3 \ln(x) \quad \text{and} \quad g(x) = (\ln(x))^3.$$

As you can see in the figure on the right, the graphs of these functions have three intersections, A , B and C .

- 5pt a Compute exactly the coordinates of the intersections A , B and C .

For all p in the interval $x_B < p < x_C$, the vertical line $x = p$ intersects the graph of f in point D_p and the graph of g in point E_p .

- 6pt b Compute exactly the value of p in the indicated interval for which the distance between points D_p and E_p is maximal and compute this maximal distance.



V is the region enclosed by the x -axis, the y -axis, the horizontal line $y = 1$ and the graph of f .

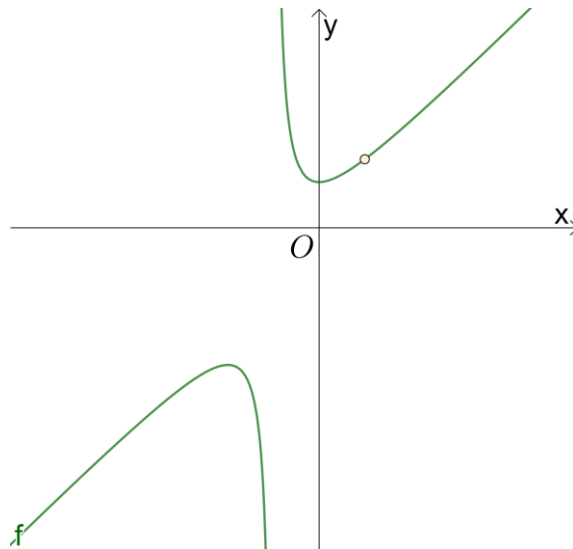
- 6pt c Compute exactly the volume of the solid of revolution that is formed by revolving V around the y -axis.

Question 5

New

In the figure below the graph is shown of the function

$$f(x) = \frac{(x-1)(x^2+x+1)}{x^2-1}$$



The graph of f has a perforation, a vertical asymptote, an oblique asymptote and two vertices, a minimum and a maximum.

- 4pt a Compute exactly the coordinates of the perforation of the graph of f .
- 2pt b Give an equation for the vertical asymptote of the graph of f .
- 4pt c Use an exact computation to determine an equation for the oblique asymptote of the graph of f .
- 5pt d Compute exactly the coordinates of the vertex where f has a maximum.

Question 6

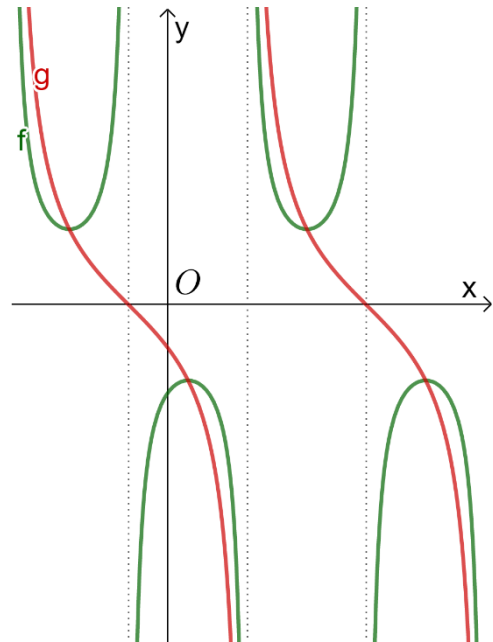
Source: CCVX entrance exam wiskunde B April 2018

In the figure on the right the graphs are shown of the functions

$$f(x) = \frac{1}{\sin\left(2x - \frac{2}{3}\pi\right)}$$

and

$$g(x) = \frac{\sin\left(\frac{5}{6}\pi - x\right)}{\cos\left(\frac{5}{6}\pi - x\right)}$$



- 4pt a Use an exact computation to find the equations of the three vertical asymptotes of the graph of f that are shown in the figure to the right.

In the figure it seems that the graphs of f and g intersect in the points where f has an extreme value (that is a minimum or a maximum).

- 6pt b Compute exactly the coordinates of the points where f has an extreme value and show that these are common points of the graphs of f and g .

Furthermore, the function h is given by $h(x) = 4 \cos\left(2x - \frac{2}{3}\pi\right)$,

- 5pt c Compute exactly the x -coordinates of the intersections of the graphs of f and h .

Extra item on the last page of this example exam.

End of the exam.

Is your name on all answer sheets?

Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t + u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t - u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t + u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t - u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2 \sin(t) \cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$$

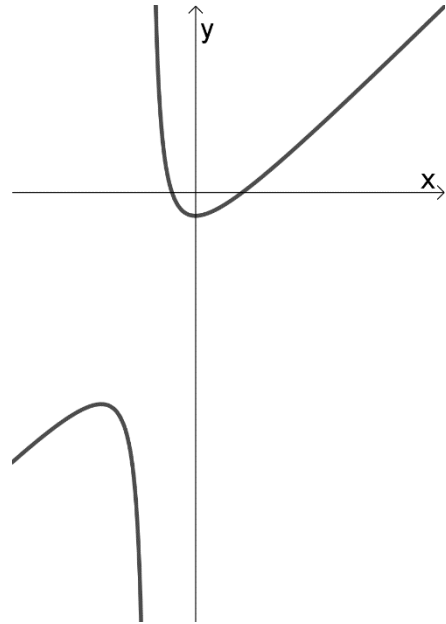
Extra question

Source: CCVX entrance exam wiskunde B April 2018

For $x \neq -2$ the function f is given by

$$f(x) = x - 3 + \frac{4}{x+2}$$

In the figure on the right a sketch of the graph of this function is shown. The intersections of the graph of f and the x -axis are $(-1,0)$ and $(2,0)$.



- 4pt a Compute exactly the x -coordinates of the intersections of the horizontal line $y = 7$ and the graph of f .
- 5pt b Compute exactly the values of p for which the horizontal line $y = p$ has no points in common with the graph of f .

Line ℓ with equation $y = -3(x + 1)$ is a tangent to the graph of f .
Line m runs parallel to line ℓ and is also a tangent to the graph of f .

- 6pt c Compute exactly an equation for line m .

V is the bounded region enclosed by the graph of f and the x -axis.

- 7pt d Compute exactly the area of region V .

Extra item for question 1

Find a vector representation for line ℓ as well,

Extra item for question 6

A is the point of the graph of g for which $x_A = \frac{2}{3}\pi$.

Compute exactly an equation for the tangent to the graph of g in point A .