CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 19 April 2019
Time: 13.30 – 16.30 hours
Questions: 6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:

Question	1	2	3	4	5	6
a	5	4	5	5	6	5
b	5	4	6	5	7	5
c	4	2	5	5		5
d		4				3
Total	14	14	16	15	13	18

$$\text{Grade} = \frac{\text{total points scored}}{10} + 1$$

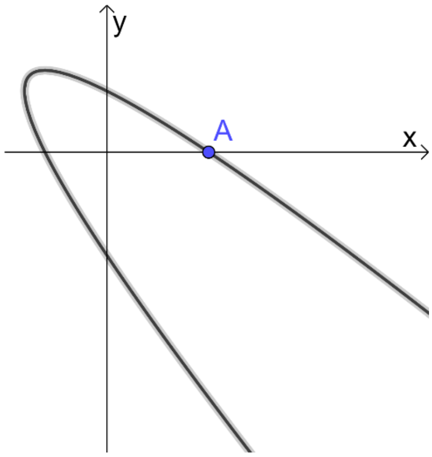
You will pass the exam if your grade is at least 5.5 .

Question 1

The movement of a point *P* is given by the parametric equations

$$\begin{cases} x(t) = t^2 - 2t - 3 \\ y(t) = -t^2 + 4 \end{cases}$$

In the figure on the right, the path of point *P* is shown. *A* is the intersection of the path of *P* with the positive *x*-axis.



5pt a Compute exactly the distance between the intersections of the path of *P* and the line l with equation $y = x + 7$.

5pt b Set up a vector representation of the tangent line to the path of *P* in *A*.

4pt c Compute exactly the minimal velocity (that is the length of the velocity vector) of point *P*.

Question 2

Circle c_1 passes through points $A(1,2)$ and $B(3,8)$. The centre *C* of circle c_1 is on the *x*-axis.

4pt a Compute the *x*-coordinate of centre *C*.

c_2 is the circle with equation $x^2 + y^2 - 2x - 4y = 0$. Line *m* is the tangent line to circle c_2 in the origin $O(0,0)$.

4pt b Compute the angle between line *m* and the positive *x*-axis.

Circle c_3 passes through the origin $O(0,0)$ and through points $D(-6,4)$ and $E(6,9)$.

2pt c Show that the vectors $\overrightarrow{OD}$ and $\overrightarrow{OE}$ are perpendicular.

4pt d Compute the coordinates of the centre of circle c_3.

Question 3

For each value of a , the function f_a is given by

$$f_a(x) = \frac{3x^3 - 3x^2 + ax}{x^2 - 4}$$

There are two values of a for which the graph of f_a has a perforation (that is a removable discontinuity).

5pt a Compute these two values of a.

In the remainder of this question, we take $a = 0$.
Furthermore, the function g is given by $g(x) = (1 - x) \cdot \mathrm{e}^{1-x}$.

6pt b Show that the graphs of f_0 and g touch in point $P(1,0)$.

5pt c Find an equation for the oblique asymptote of f_0 .

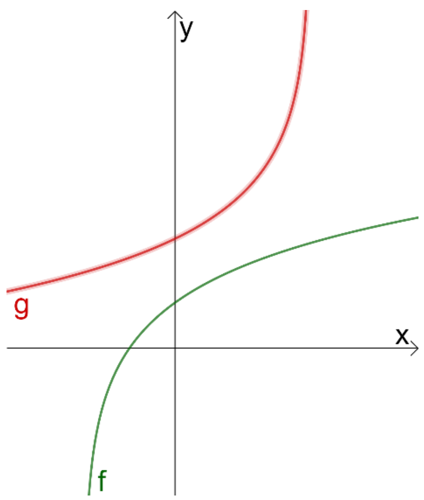
Question 4

In the figure on the right, the graphs are shown of the functions

$$f(x) = {}^{2}\log(x + 2)$$

and

$$g(x) = {}^{2}\log\left(\frac{16}{3 - x}\right)$$



For each p in the common domain of f and g, the points F_p and G_p are the intersections of the vertical line $x = p$ and the graphs of f and g respectively.

5pt a Compute the value(s) of p for which the distance between the points F_p and G_p equals 2.

5pt b Compute the value(s) of p for which the distance between the points F_p and G_p is minimal.

Furthermore, the function h is given by $h(x) = {}^{4}\log(x^2 + 4x + 4)$.

5pt c Find the values of x for which $f(x) = h(x)$.

Question 5

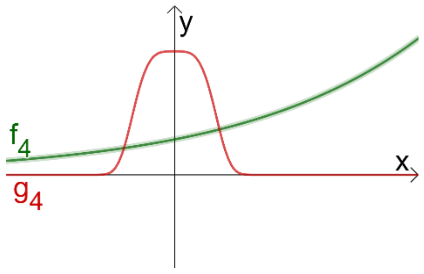
For each positive integer a, the functions f_a and g_a are given by

$$f_a(x) = \exp\left(\frac{x-1}{a}\right)$$

and

$$g_a(x) = \exp(1 - x^a)$$

N.B.: $\exp(X) = \mathrm{e}^X$



In the figure on the right, the graphs are shown of the functions f_4 and g_4.

6pt a Show that for each positive integer a the graphs of f_a and g_a intersect at a right angle at point $S(1,1)$.

For each $p > 1$, V_p is the region enclosed by the line $x = 1$, the line $x = p$, the graph of f_4 and the *x*-axis, and S_p is the solid of revolution that is formed by revolving V_p round the *x*-axis.

7pt b Compute the value of p for which the volume of S_p equals 2π .

Question 6

Given are the functions $f(x) = 2\cos^2(x) + \cos(x) - 1$ and $g(x) = \cos^2(x)$.

5pt a Compute the *x*-coordinates of the intersections of the graph of f and the *x*-axis on the interval $0 \le x \le 2\pi$.

5pt b Show that $G(x) = \frac{1}{2}x + \frac{1}{4}\sin(2x)$ is an antiderivative of $g(x)$.

5pt c Compute $\int_0^{\frac{\pi}{2}} f(x)\,\mathrm{d}x$.

Furthermore are given the functions $h(x) = \cos(5x)$ and $k(x) = \cos\left(5x - \frac{1}{4}\pi\right)$.

3pt d Find the number of intersections of the graphs of h and k on the interval $0 \le x \le \pi$.

End of the exam.

Is your name on all answer sheets?

Formula list wiskunde B

$\sin^2(x) + \cos^2(x) = 1$

$\sin(t + u) = \sin t \cos u + \cos t \sin u$

$\sin(t - u) = \sin t \cos u - \cos t \sin u$

$\cos(t + u) = \cos t \cos u - \sin t \sin u$

$\cos(t - u) = \cos t \cos u + \sin t \sin u$

$\sin(2t) = 2 \sin(t) \cos(t)$

$\cos(2t) = \cos^2(t) - \sin^2(t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$

