

CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 15 December 2020
Time: 140 minutes (2 hours and 20 minutes) (Due to corona circumstances this exam has been shortened to 140 minutes)
Questions: 5

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Since the time for this exam has been reduced to 140 minutes (instead of 180), the number of questions is reduced. Therefore, the total number of points that can be scored is reduced to 70 (instead of 90).

Points that can be scored for each item:					
Question	1	2	3	4	5
a	6	4	7	6	4
b	6	3	5	6	7
c		4		2	
d		5		5	
Total	12	16	12	19	11
Grade = $\frac{\text{total points scored}}{10} \times \frac{9}{7} + 1$					
You will pass the exam if your grade is at least 5.5 .					

Question 1

Take a new answer sheet for every question!

Given are the function

$$f(x) = \frac{x^2 - 2x - 8}{x^2 - 4}$$

and the family of functions

$$g_{ab}(x) = ax^2 + b$$

- 6pt a Compute exactly the values of a and b for which the graphs of f and g_{ab} are touching at point $A(1, 3)$.

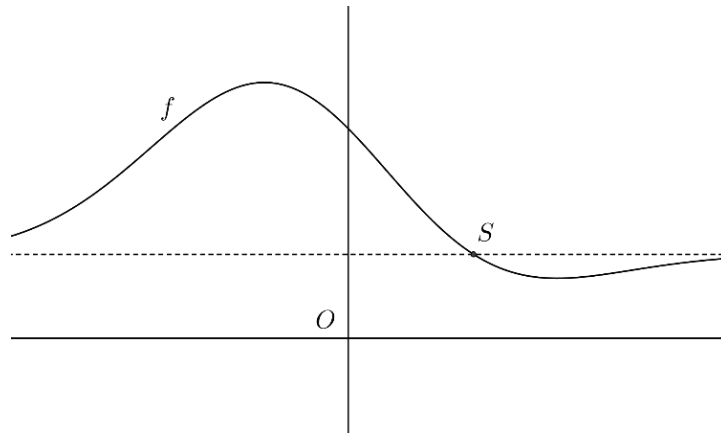
The graph of f has one perforation (that is a removable discontinuity).

- 6pt b Compute exactly the coordinates of the perforation of the graph of f .

Question 2

Take a new answer sheet for every question!

In the figure below, the graph is shown of the function $f(x) = 2 + (3 - x)e^{-0.05x^2}$. Also shown is the horizontal asymptote of this graph.



Point S is the intersection of the graph of f and its horizontal asymptote.

4pt a Compute exactly the coordinates of point S.

The derivative of f is $f'(x) = (0.1x^2 - 0.3x - 1)e^{-0.05x^2}$.

3pt b Show that this is true.

As you can see, the function f has two extremes, a maximum and a minimum.

4pt c Compute exactly the coordinates of the two points on the graph of f at which the function has an extreme.

Line k with vector representation $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -7 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 1 \end{pmatrix}$ intersects the graph of f in a point on the vertical axis.

5pt d Compute algebraically the angle between line k and the graph of f in this intersection. Give your answer in degrees.

Question 3

Take a new answer sheet for every question!

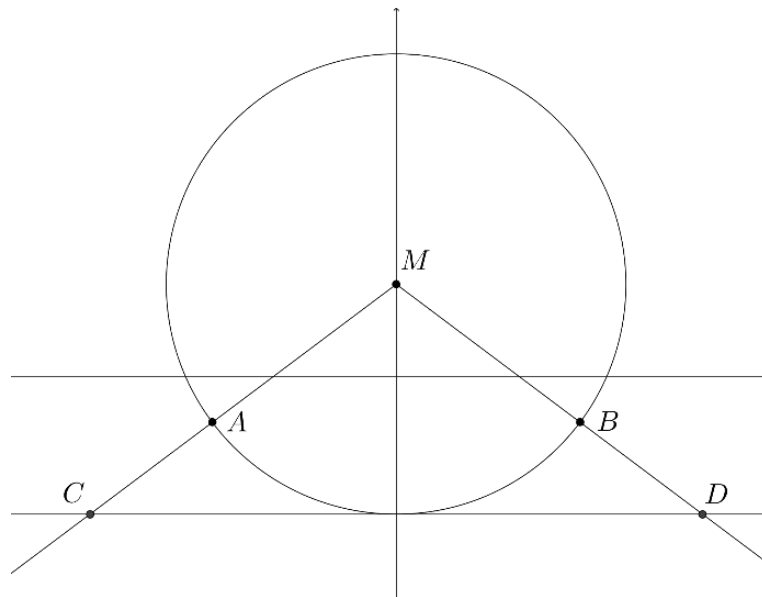
Given is the circle with centre M and equation $x^2 + (y - 2)^2 = 25$.

Points A and B are the points on this circle with y -coordinate -1 .

Point C is the intersection of the line through M and A and the horizontal tangent line to the circle that lies below the x -axis.

Point D is the intersection of the line through M and B and this horizontal tangent line.

See the figure below.



7pt a Compute exactly the area of triangle MCD .

Furthermore, the point $N(9, 14)$ is given.

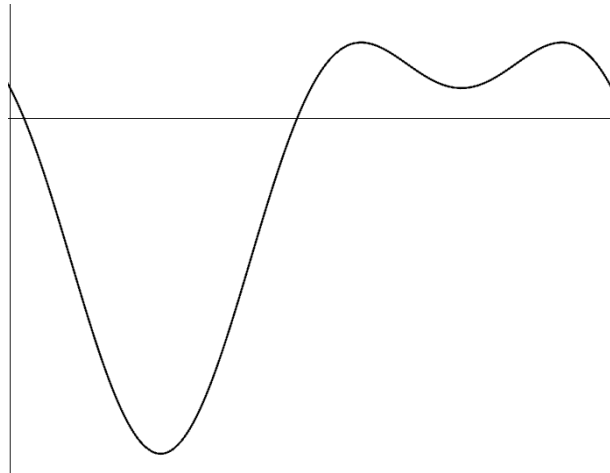
There are two circles with centre N that touch the circle given above.

5pt b For both of these circles, exactly compute the radius.

Question 4

Take a new answer sheet for every question!

For $0 \leq x \leq 2\pi$, the function f is given by $f(x) = 6 \cos^2(x) - 6 \sin(x) - 5$.
In the figure below, a sketch of the graph of f is shown.



6pt a Compute exactly the distance between the two points where the graph of f has a maximum.

6pt b Compute exactly the values of x for which $f(x) = -3\frac{1}{2}$.

The function rule for f can also be written as

$$f(x) = 3 \cos(2x) - 6 \sin(x) - 2$$

2pt c Show that this is true.

V is the region enclosed by the graph of f , the x -axis and the vertical lines $x = \pi$ and $x = 2\pi$.

5pt d Compute the area of region V exactly.

Question 5

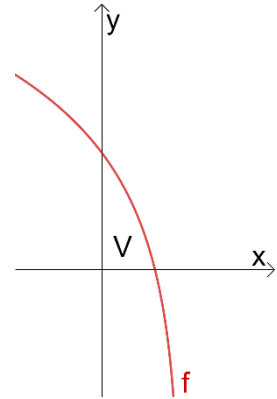
Take a new answer sheet for every question!

Given are the functions $f(x) = \ln(3 - 4x)$ and $g(x) = 2 \ln(x) + \ln(4)$.

4pt a Solve exactly: $f(x) = g(x)$.

In the figure on the right, V is the region enclosed by the graph of f , the x -axis and the y -axis.

7pt b Compute exactly the volume of the solid of revolution that is formed by rotating V round the y -axis.



End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*

- empty sheets, please leave them on your table;*
- sheets with only your name on it, please take them with you;*
- scrap paper;*
- these questions.*

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sin(t + u) = \sin t \cos u + \cos t \sin u$$

$$\sin(t - u) = \sin t \cos u - \cos t \sin u$$

$$\cos(t + u) = \cos t \cos u - \sin t \sin u$$

$$\cos(t - u) = \cos t \cos u + \sin t \sin u$$

$$\sin(2t) = 2 \sin(t) \cos(t)$$

$$\cos(2t) = \cos^2(t) - \sin^2(t) = 2 \cos^2(t) - 1 = 1 - 2 \sin^2(t)$$