CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date: 17 April 2021
Time: 140 minutes (2 hours and 20 minutes)
Questions: 5

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Because the time for this exam has been reduced to 140 minutes, the number of questions has been reduced. Therefore, the total number of points that can be scored is reduced to 72.

Points that can be scored for each item:					
Question	1	2	3	4	5
a b c	6 5 5	5 5	7 4 5	5 7 7	5 6
Total	16	10	16	19	11
Grade $= \frac{\text{total points scored}}{8} + 1$ You will pass the exam if your grade is at least 5.5 .					

Question 1

Take a new answer sheet for every question!

Given are the function

$$f(x) = (2x - 3) \cdot \mathrm{e}^{2x-1}$$

and the family of functions

$$g_{ab}(x) = x^2 + ax + b$$

6pt a Compute exactly the values of a and b for which the graphs of f and g_{ab} are touching at point $A\left(\frac{1}{2}, -2\right)$.

The graph of f and the line $y = 6x - 9$ intersect in two points.
The *x*-coordinate of one of these points, x_1, is a rational number.
The *x*-coordinate the other point can be written as $x_2 = \ln\sqrt{c}$.

5pt b Compute exactly the values of x_1 and c.

The following transformations are successively applied to the graph of f

- a reflexion in the *x*-axis
- a multiplication with factor 2 with respect to the *x*-axis
- a translation over the vector $\begin{pmatrix}1\\3\end{pmatrix}$

to get the graph of a function h.

5pt c Find a function rule $h(x) = \cdots$ for this function h. Explain your answer.

Question 2

Take a new answer sheet for every question!

Given is the family of functions

$$f_a(x) = \frac{x^3 - 2x^2}{x^2 + a}$$

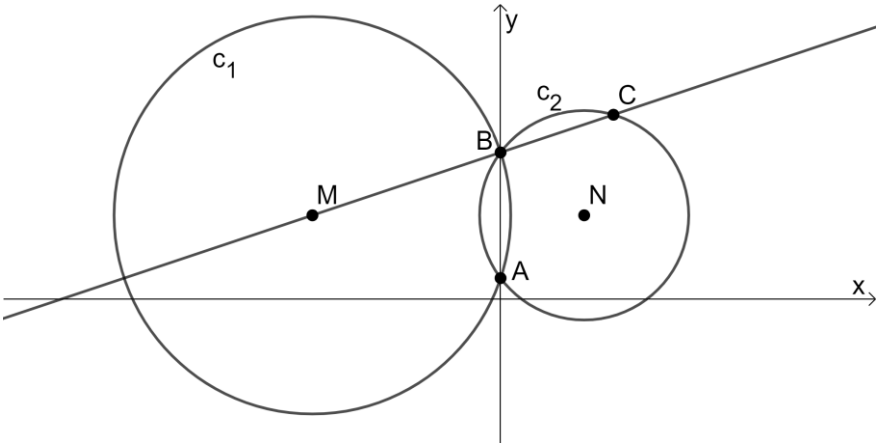
5pt a Compute exactly the values of a for which the graph of f_a has two vertical asymptotes.

5pt b Use an exact computation to find an equation for the oblique (= *slant*) asymptote of the function f_1.

Question 3

Take a new answer sheet for every question!

Given are circle c_1 with centre M and equation $(x+9)^2+(y-4)^2=90$ and circle c_2 with centre $N(4,4)$ and radius $r=5$.
These circles intersect in points A and B on the *y*-axis.
The straight line through M and B intersects circle c_2 in point C.
See the figure below.

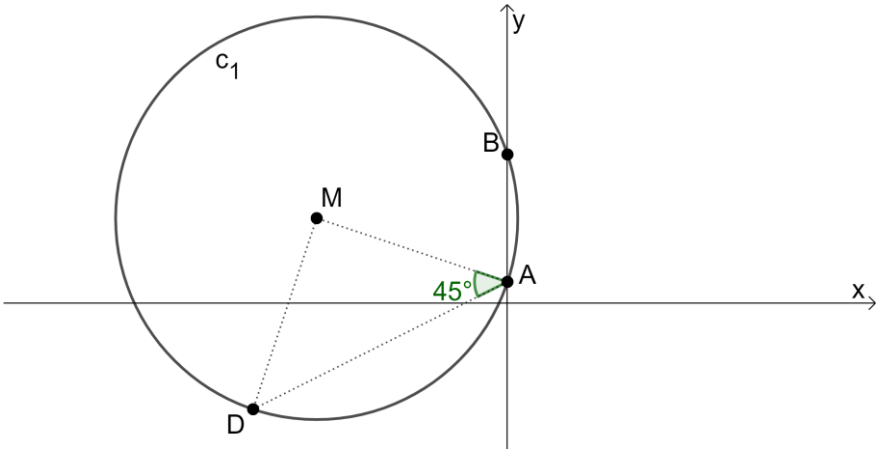


7pt a Compute exactly the *x*-coordinate of point C.

There are two points (P and Q) on the vertical line $x=-1$ for which the distance to circle c_2 is equal to 8.

4pt b Compute exactly the area of triangle NPQ.

In the figure below, D is a point on the circle c_1 given above, such that in triangle MAD we have $\angle MAD=45°$.



5pt c Compute exactly a vector representation for the line through M and D.

Question 4

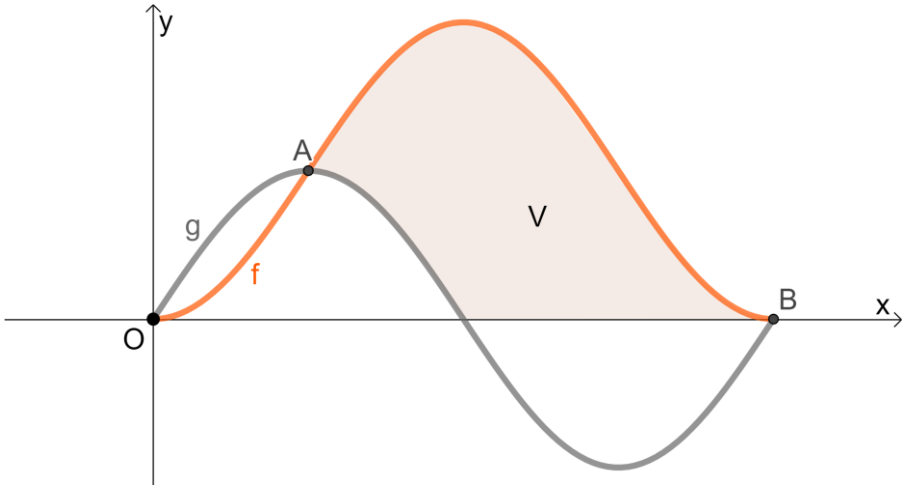
Take a new answer sheet for every question!

For $0 \leq x \leq \frac{2}{3}\pi$, the functions f and g are given by

$$f(x) = 1 - \cos(3x) \text{ and } g(x) = \sin(3x) .$$

The points $O(0,0)$, $A\left(\frac{1}{6}\pi, 1\right)$ and $B\left(\frac{2}{3}\pi, 0\right)$ are the intersections of the graphs of f and g on the given interval, see the figure below.

In this figure, a region *V*, enclosed by the *x*-axis and the graphs of f and g, is shaded.



5pt a Compute algebraically the angle at which the graphs of f and g intersect in point A. Give your answer in degrees.

7pt b Compute exactly the area of region *V*.

Furthermore, the function h is given by $h(x) = \cos\left(6x - \frac{1}{3}\pi\right)$.

7pt c Compute exactly the *x*-coordinates of the intersections of the graphs of g and h on the interval $0 \leq x \leq \frac{2}{3}\pi$.

Question 5

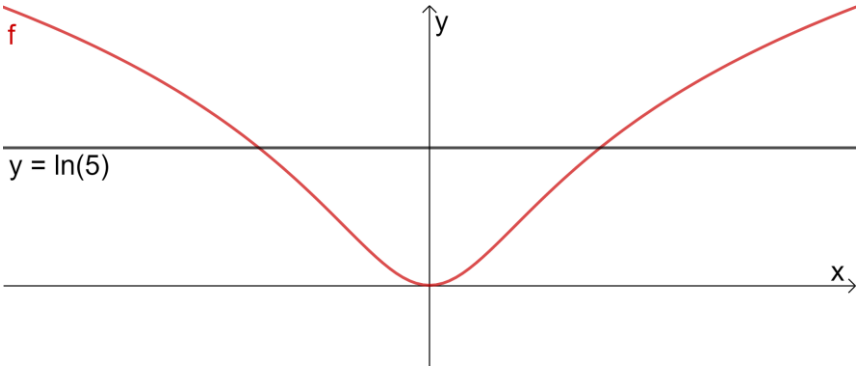
Take a new answer sheet for every question!

The functions f and g are given by

$$f(x) = \ln(x^2 + 1) \text{ and } g(x) = 2\ln(2x - 1) .$$

5pt a Solve exactly: $f(x) = g(x)$.

In the figure below, *V* is the region enclosed by the graph of f and the horizontal line $y = \ln(5)$.



6pt b Compute exactly the volume of the solid of revolution that is formed by rotating *V* round the *y*-axis.

End of the exam.

*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

*What should **not** be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$\sin^2(x) + \cos^2(x) = 1$

$\sin(t+u) = \sin t \cos u + \cos t \sin u$

$\sin(t-u) = \sin t \cos u - \cos t \sin u$

$\cos(t+u) = \cos t \cos u - \sin t \sin u$

$\cos(t-u) = \cos t \cos u + \sin t \sin u$

$\sin(2t) = 2\sin(t)\cos(t)$

$\cos(2t) = \cos^2(t) - \sin^2(t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$

